task 4. 计算球状星团、椭圆星系、星系团中的 σ、t_{cross}、t_{relax}.

- 位力定理 $2\langle KE\rangle + \langle PE\rangle = 0 \Rightarrow \sigma \sim \sqrt{\frac{GM}{r}}$ $\quad G = 4.3\times10^{-6}\ \mathrm{kpc}\cdot M_\odot^{-1}\cdot(\mathrm{km/s})^2$
- $t_{cross} \sim \frac{r}{v}$
- $\frac{t_{relax}}{t_{cross}} \sim \frac{v^4R^2}{6NG^2m^2\ln\Lambda} \sim \frac{N}{6\ln(N/2)}$

(a) globular cluster: $M_{gc}\sim10^5 M_\odot$, $r\sim0.1$ kpc, $N\sim10^5$

$\sigma \sim \sqrt{\frac{GM}{r}} \sim$ (2 km/s)

$t_{cross} \sim \frac{r}{v} \sim$ (5×10^7 yr)

$t_{relax} \sim \frac{N}{6\ln(N/2)} \sim$ (5×10^{10} yr)

(b) elliptical galaxy: $M_g\sim10^{11}M_\odot$, $r\sim100$ kpc, $N\sim10^{10}$

$\sigma \sim$ (60 km/s)

$t_{cross} \sim$ (2×10^9 yr)

$t_{relax} \sim$ (10^{19} yr)

(c) galaxy cluster: $M_c\sim10^{14}M_\odot$, $r\sim5$ Mpc, $N_{galaxies}\sim10^3$

$\sigma \sim$ (300 km/s)

$t_{cross} \sim$ (2×10^{10} yr)

$t_{relax} \sim$ (5×10^{11} yr)

task 5. 估算银河系角动量自旋参数 λ.

$$\lambda = J\,|E|^{1/2}\,G^{-1}\,M^{-5/2}$$

$$E = -\frac{GM^2}{2r_{200}} = -\frac{MV_c^2}{2}$$

$$J = \frac{Jd}{jd} \sim \frac{2M_d R_d V_c}{m_d} \sim \frac{2M_d R_d V_c}{M_d} \cdot M$$

$$\rightarrow J \sim 2R_d V_c M$$

$$\Rightarrow \lambda \sim 2R_d V_c M \cdot \left(\frac{MV_c^2}{2}\right)^{1/2} \cdot \frac{1}{G} \cdot M^{-\frac{5}{2}}$$

$$\lambda \sim \frac{\sqrt{2}\, R_d \cdot V_c^2}{GM} \sim 0.03$$