星系天文学《二》：星系的统计性质

沈世银

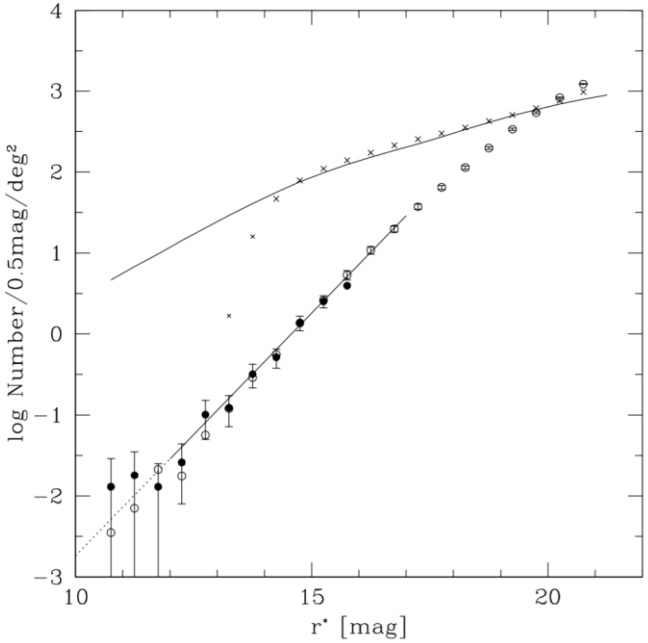
目录

- **星系性质的分布函数**
  - 星等、光度、质量
    - 不同波段
    - 不同质量成份
  - 空间分布
- **条件分布函数**

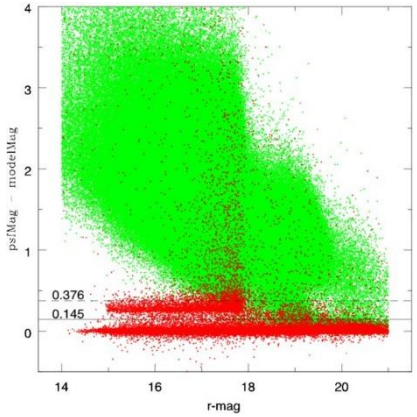
- **标度律（scaling relation）**

（一）星系性质的分布函数
1、星系的数目分布（number counts）

- N(m): 单位立体角里面单位星等星系的数目
- 如果欧几里得空间，星系的数密度是常数
  - Log N(m)= 0.6 m + const.
  - 奥博斯佯谬

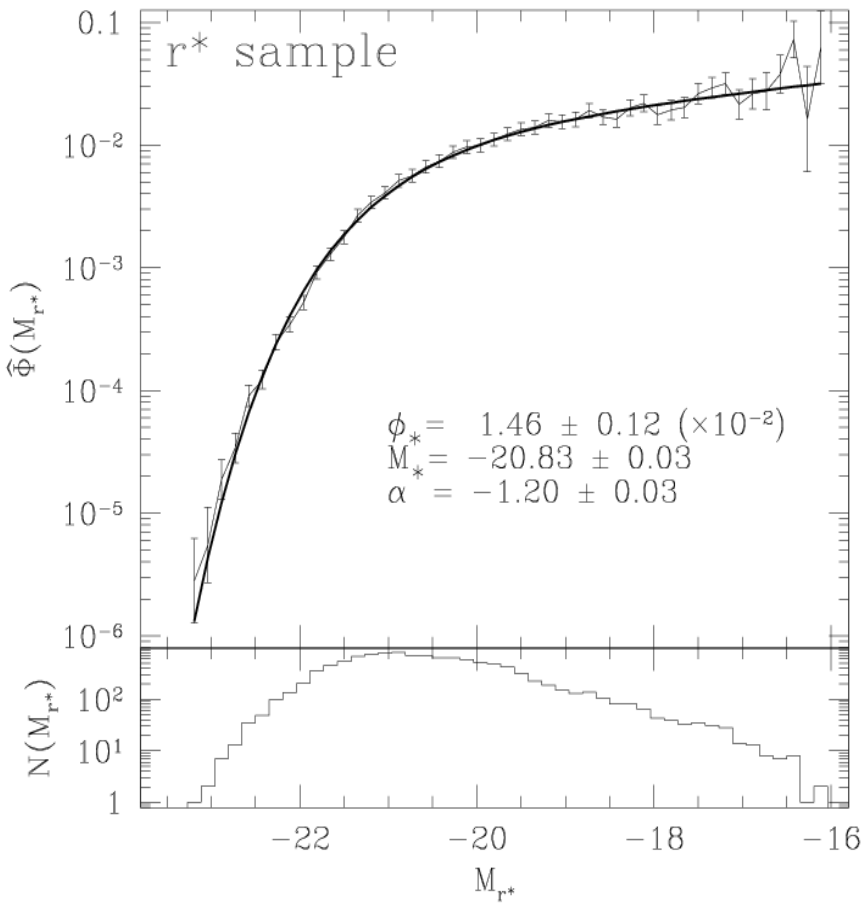


- 星系计数
  - 星系光度函数及演化
  - 宇宙学
  - 银河系尘埃消光



2、星系的光度函数（luminosity function）

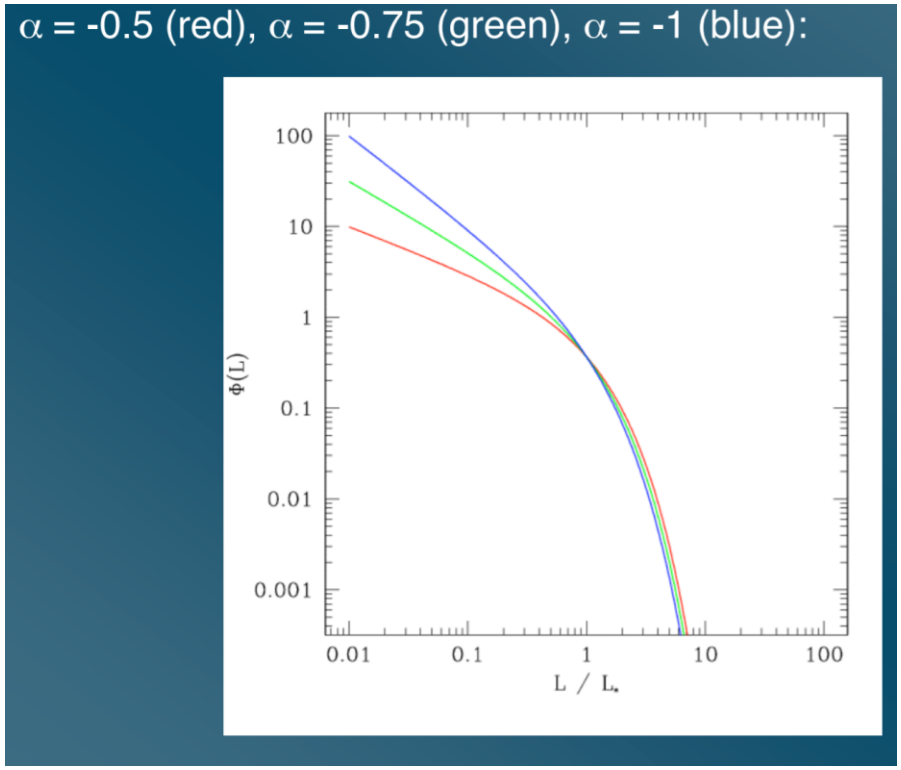
- **星系的光度函数**
  - 最基本的分布函数
  - 单位体积单位光度（星等）范围内星系的数目
    - 暗星系多，亮星系少
  - 观测的数目？多？少



Schechter 函数

$$\Phi(L)\dot{L} = \Phi_*\left(\frac{L}{L_*}\right)^{\alpha}\exp(-L/L^*)\frac{\dot{L}}{L_*}$$

- Φ_* （光度密度）
- α （暗段斜率）
- L_* （特征光度）



- 星系的数密度
  - 如果α <-1, n 发散
- 星系的光度密度
  - 由L_* 星系主导

$$n = \int_L^{\infty} \Phi(L)dL$$

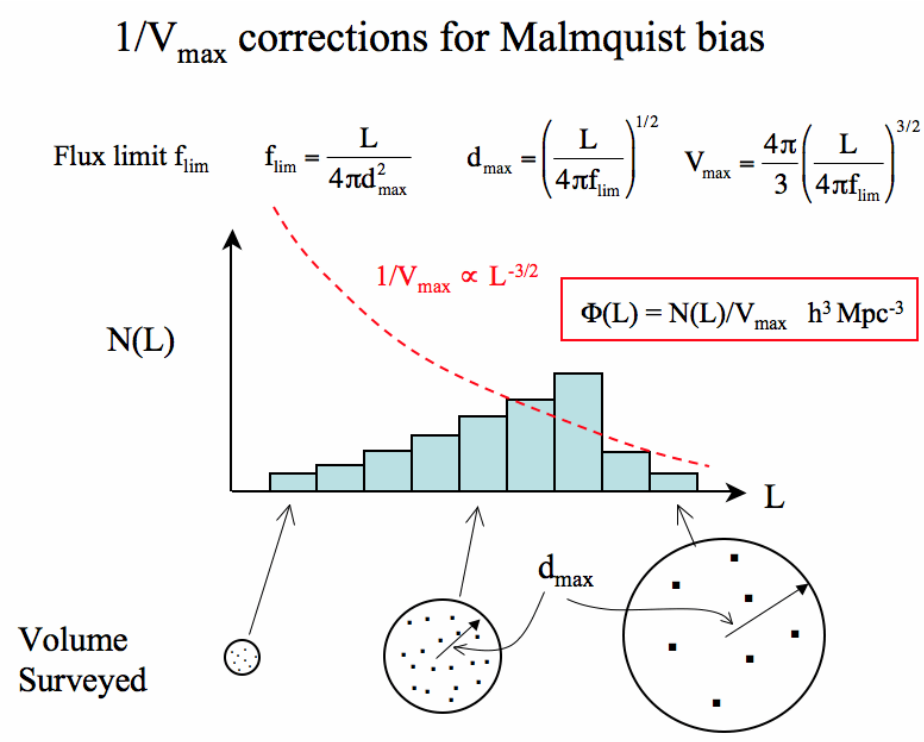
$$\int_L^{\infty} \Phi(L)LdL$$

$$\phi(M) = \frac{\ln 10}{2.5}\phi^*\left(10^{0.4(M^*-M)}\right)^{(\alpha+1)}\exp\left[-10^{0.4(M^*-M)}\right]$$

光度函数的计算

- 体积完备样本

- 星等完备样本
  - SDSS主星系样本 r<17.77
  - 亮星系更容易被观测到
  - Vmax：
    - 星系在星等完备样本中能观测到的最大体积范围
    - <V/Vmax>~0.5



最大似然法

- https://core.ac.uk/download/pdf/25215650.pdf
- Sandage, Tammann & Yahil (1979): STY

$$p(M_i, z_i) = \frac{\phi(M_i)}{\int_{-\infty}^{M_{faint(z_i)}} \phi(M) dM}.$$

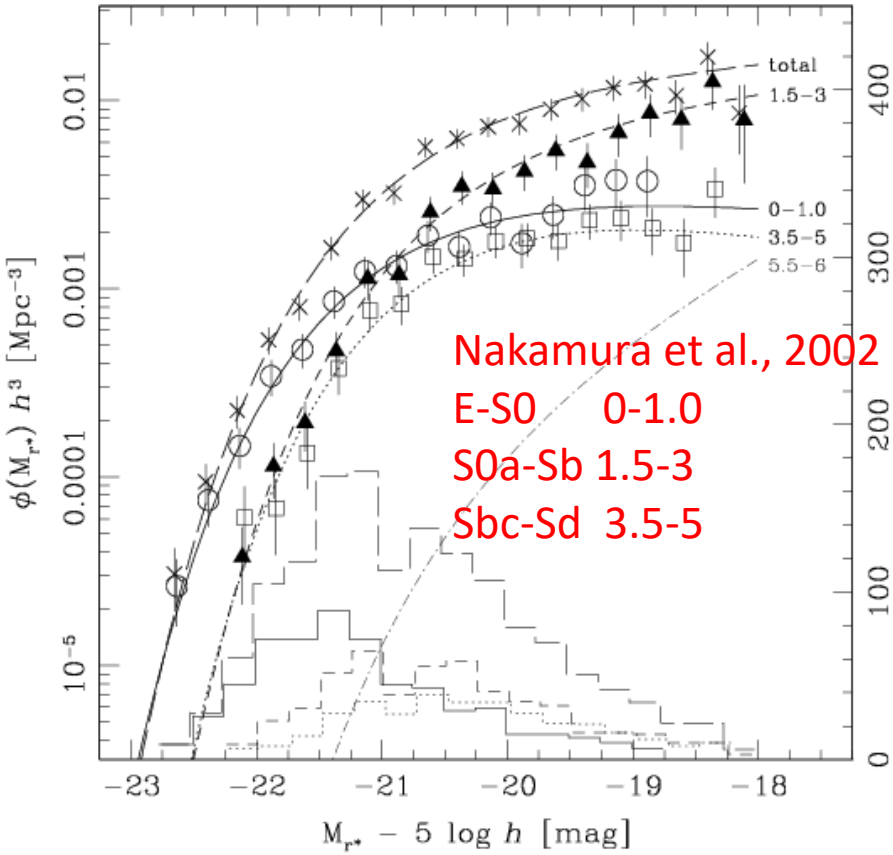
$$\mathcal{L} = \prod_{i=1}^{N_g} p(M_i, z_i).$$

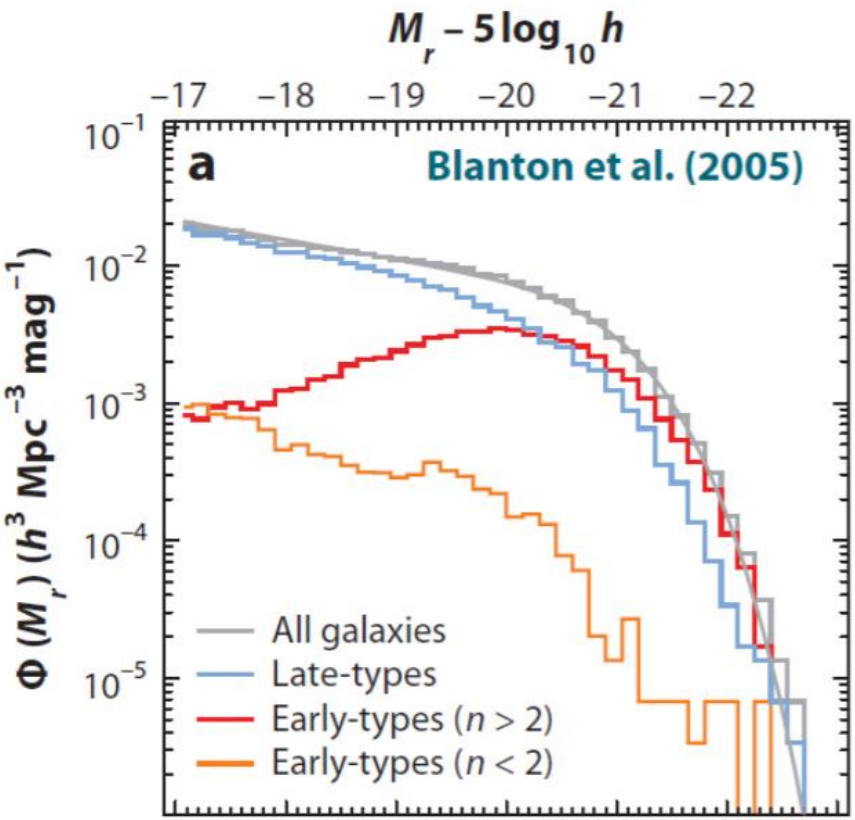
- Efstathiou, Ellis & Peterson (1988, EEP) derived the stepwise maximum likelihood method (SWML)
  - 非参数化的光度函数形式

光度函数计算中的细节

- K改正
- 演化效应改正
- 完备性改正:
  - 低表面亮度星系

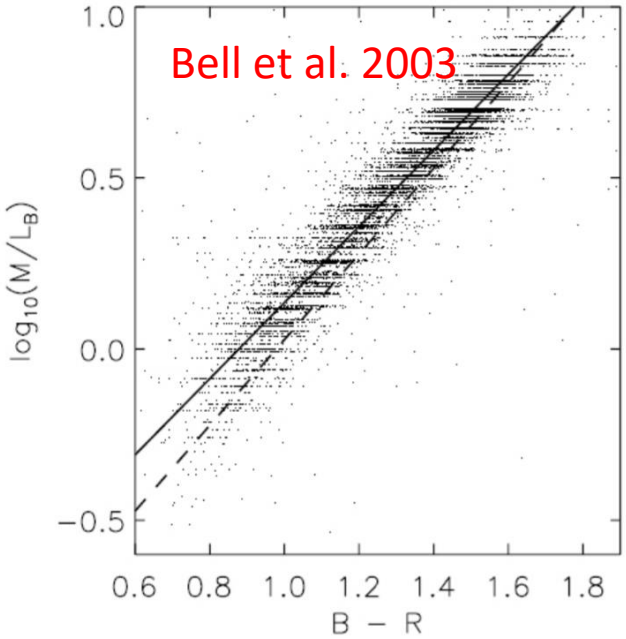
子样本的光度函数

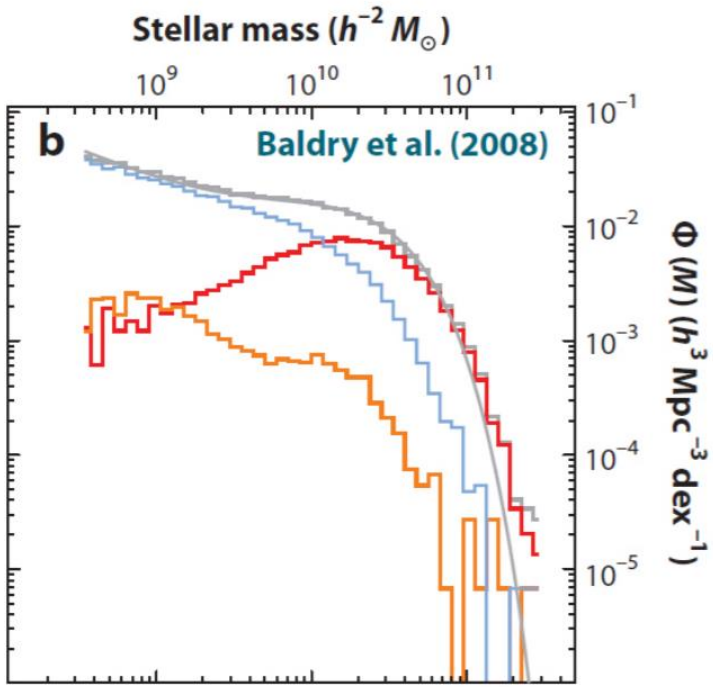

total
1.5–3
0–1.0
3.5–5
5.5–6
Nakamura et al., 2002
E-S0 0-1.0
S0a-Sb 1.5-3
Sbc-Sd 3.5-5
$\phi(M_{r*})\ h^3$ [Mpc^{-3}]
$M_{r*} - 5 \log h$ [mag]

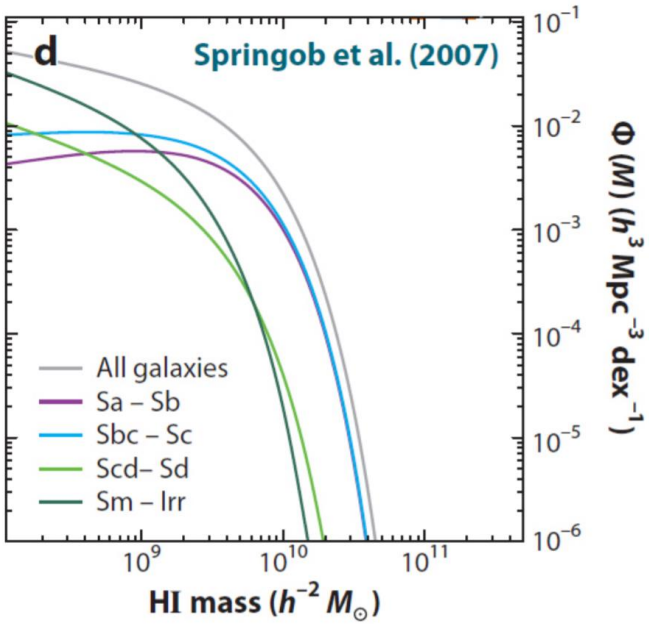

$M_r - 5\log_{10} h$
a
Blanton et al. (2005)
All galaxies
Late-types
Early-types ($n > 2$)
Early-types ($n < 2$)
$\Phi(M_r)$ (h^3 Mpc^{-3} mag^{-1})

2.1 质量函数

- Φ(M)
  - 恒星质量，气体质量等
  - 质光比：从颜色推算
    - 星族，IMF

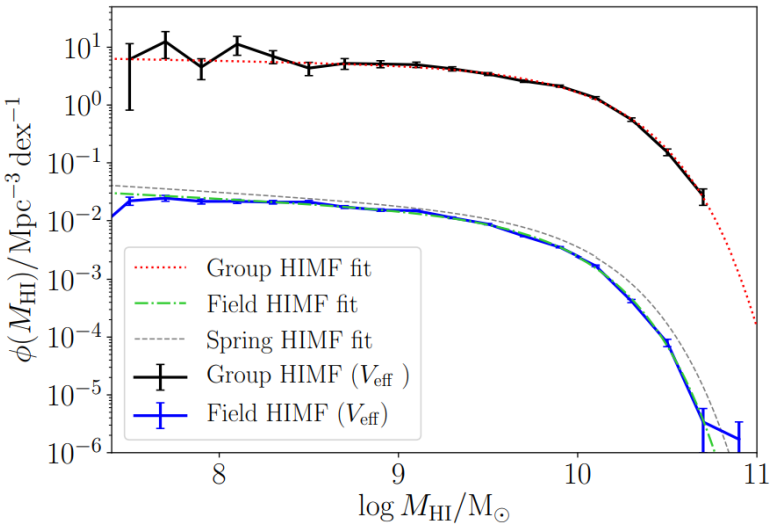




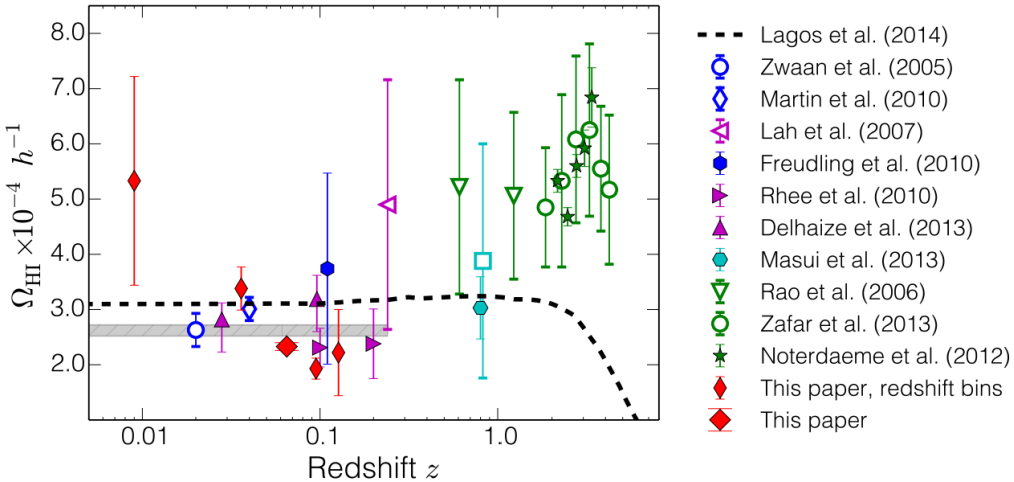


HI mass function

Jones et al. 2020

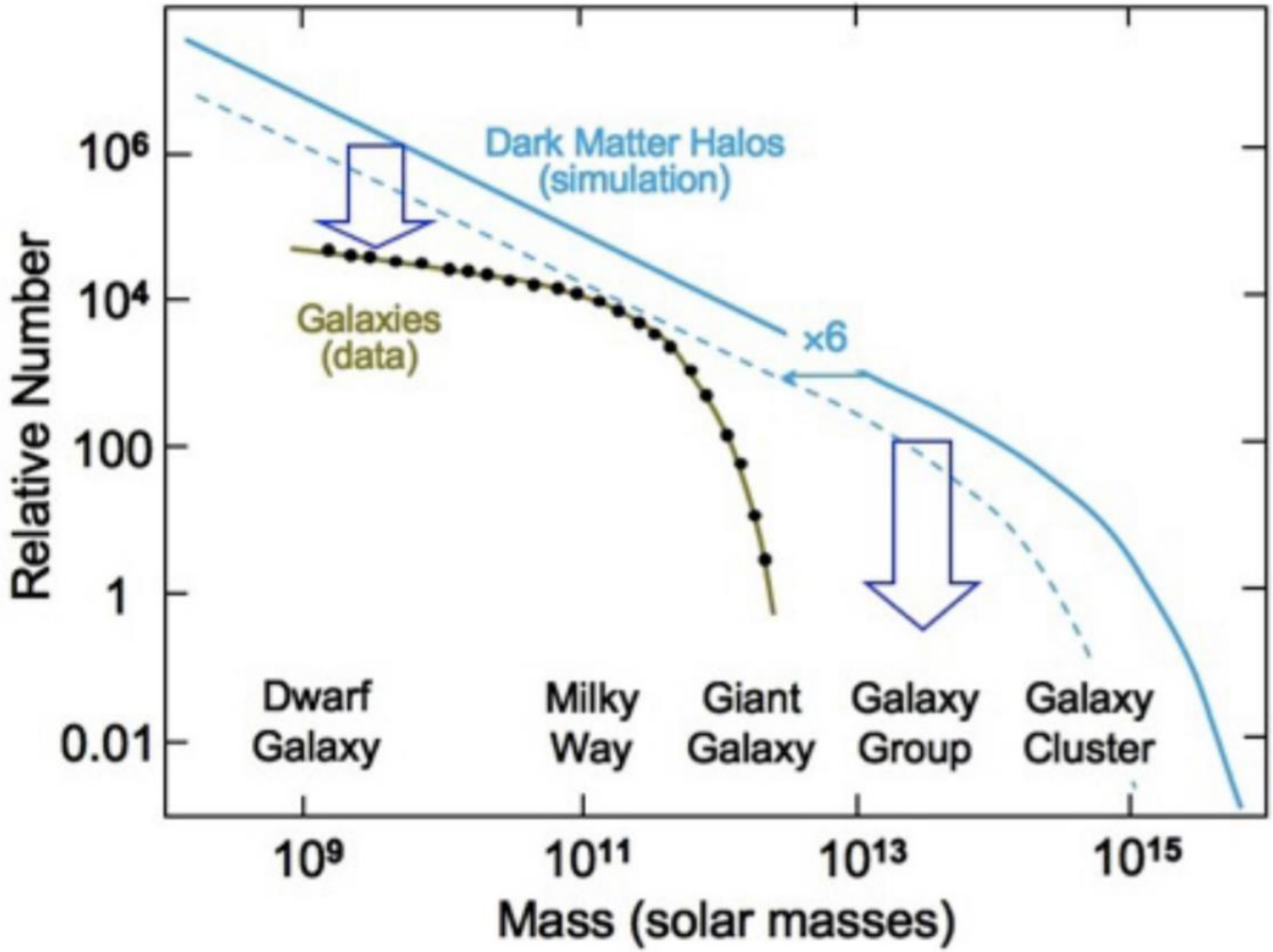


Hoppmann et al. 2015



结合HI和恒星质量函数可以更好的理解星系中的恒星形成过程

Halo and Galaxy Mass Distributions



作业1

- 单位平方度内，有多少r<20.5 mag 的星系？

- SDSS给出的本地星系的Schechter光度函数的参数为M_r*=-20.83，α=-1.0，请问单位体积范围内M_r=-16 mag的星系数目是M_r=-22mag星系数目的多少倍？SDSS是星等极限样本（r<17.77）,请问在SDSS的观测样本中， M_r=-22 mag的星系数目是M_r=-16mag星系数目的多少倍？

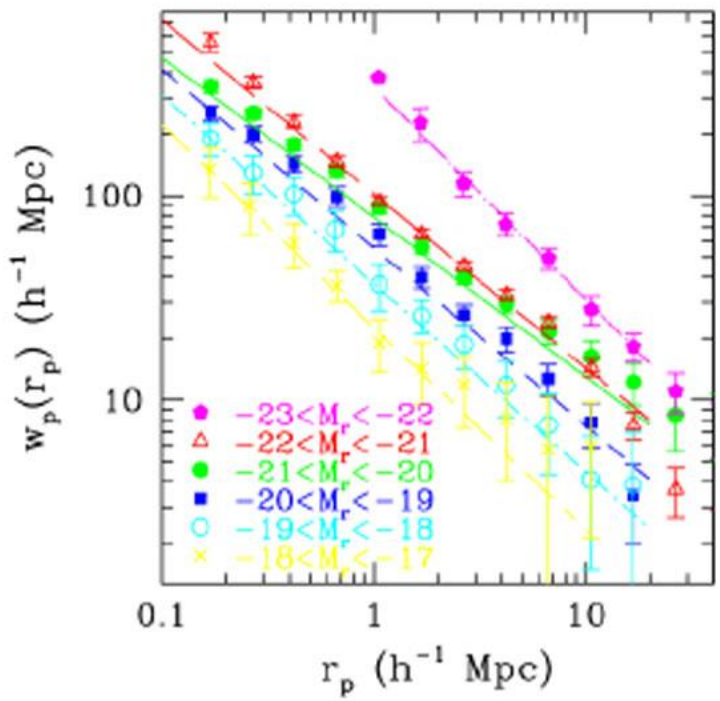
3. 星系的空间分布 http://arxiv.org/abs/1202.6633

- 两点相关函数
  - 三维 $dP_{12} = \bar{n}^2 \{1 + \xi_{12}(r)\} dV_1$
  - 二维 $dP = N[1 + \omega(\theta)]d\Omega$

$$\hat{\xi}_{DP}(r) = \frac{N_R}{N_D}\frac{DD(r)}{DR(r)} - 1$$

Davis and Peebles (1983)

- 幂率函数 $\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$
  - r0：相关长度
  - 参数和样本（光度/质量）相关
- 自相关，交相关



红移畸变（redshift distortion）

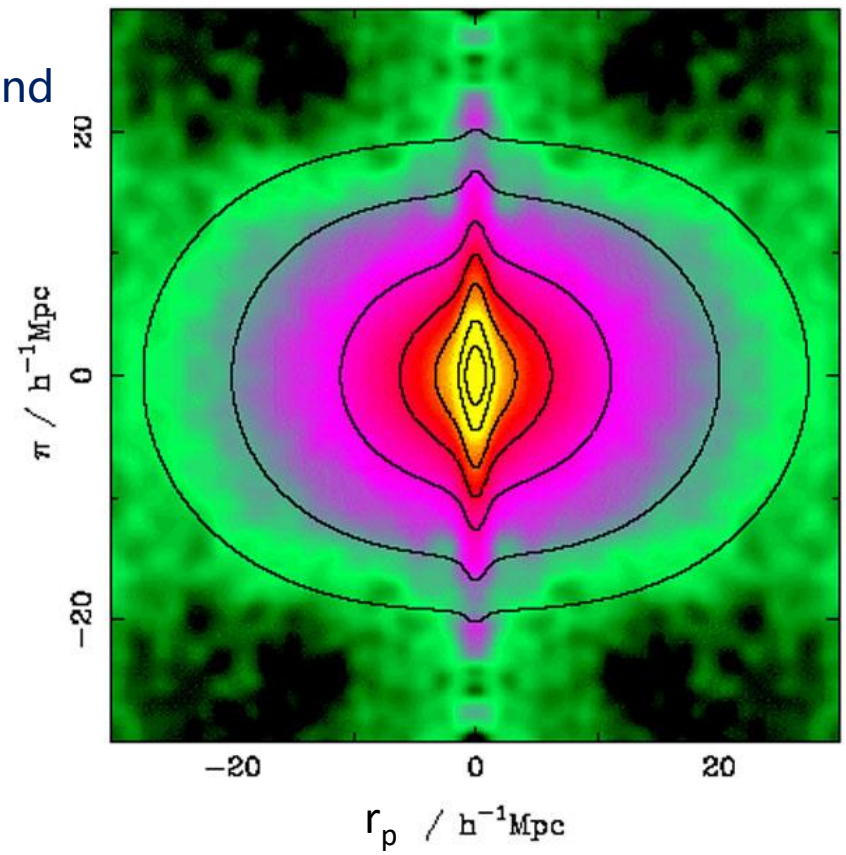
$\mathbf{v_1}$ and $\mathbf{v_2}$ are defined to be the redshift positions of a pair of galaxies

- $\mathbf{s}=(\mathbf{v_1} - \mathbf{v_2})$ to be the redshift space separation
- $\mathbf{l} = 1/2\ (\mathbf{v_1} + \mathbf{v_2})$ to be the mean distance to the pair

Separation betweentwo galaxies across (r_p) and along (π) the line of sight

$$\pi = \frac{s \cdot l}{|l|},$$

$$r_p = \sqrt{s \cdot s - \pi^2}.$$

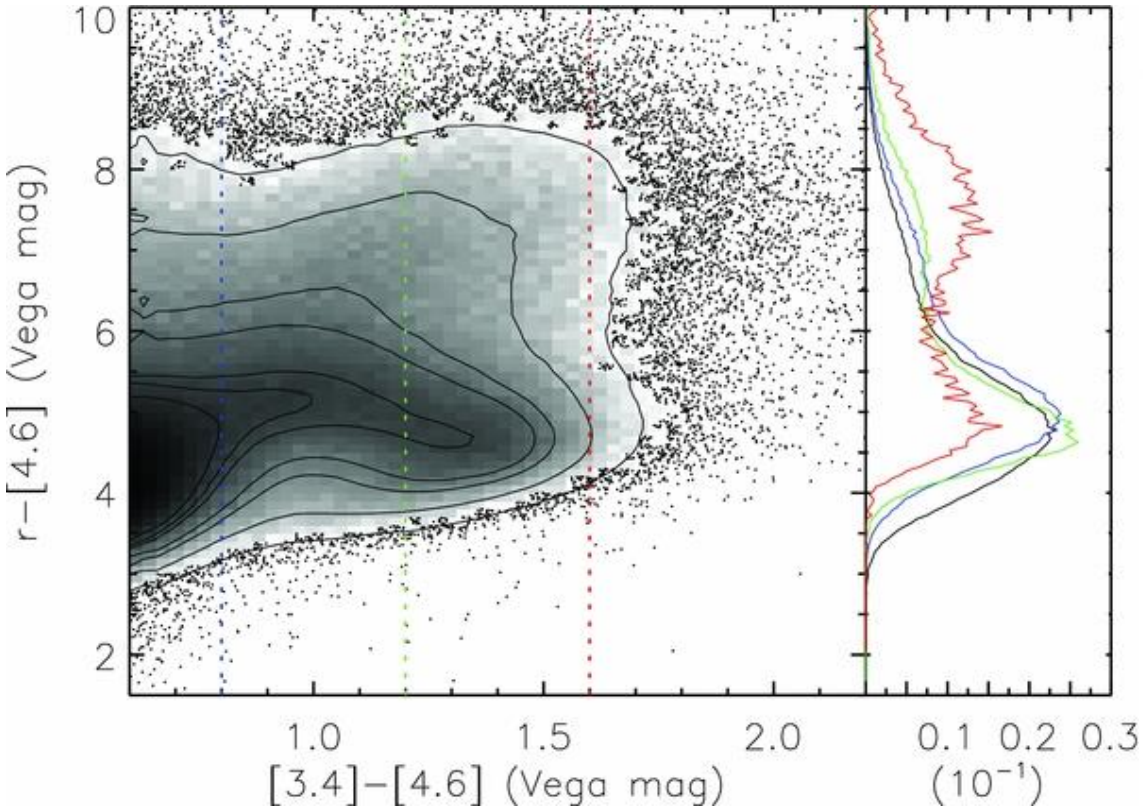


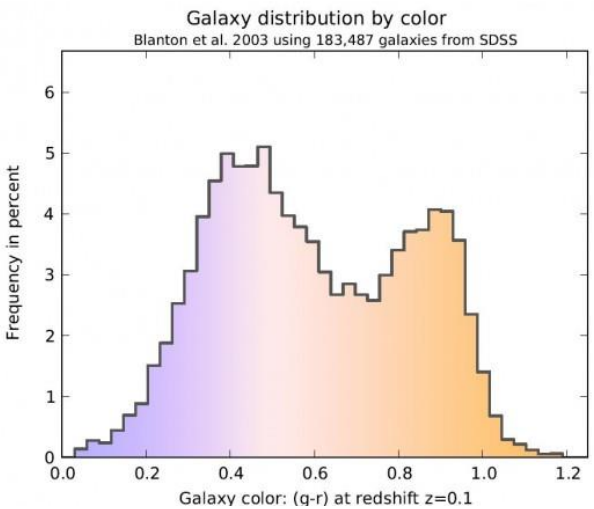
- 小尺度
  - Finger of God

    星系团中星系的随机运动
- 大尺度
  - Kaiser effect

    大尺度上星系的引力成团效应

4、星系的颜色分布





为什么是双峰分布？

（二）星系的条件分布函数

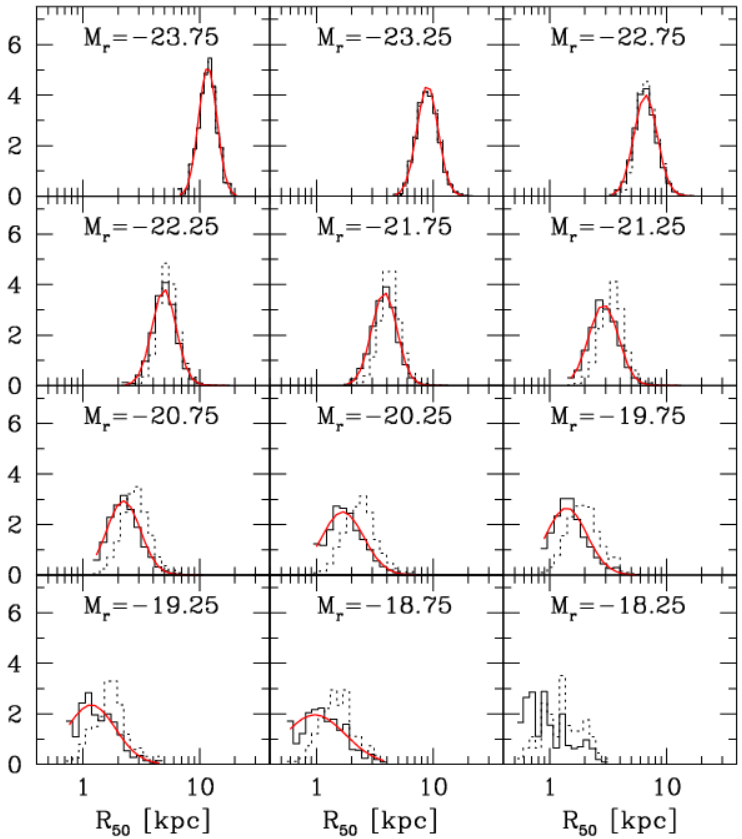
- 第一参量：质量
  - 可以是恒星质量 （观测研究）
  - 也可以是暗晕质量 （模型研究）

- 其它参量
  - f(R|M)
  - f(color|M)

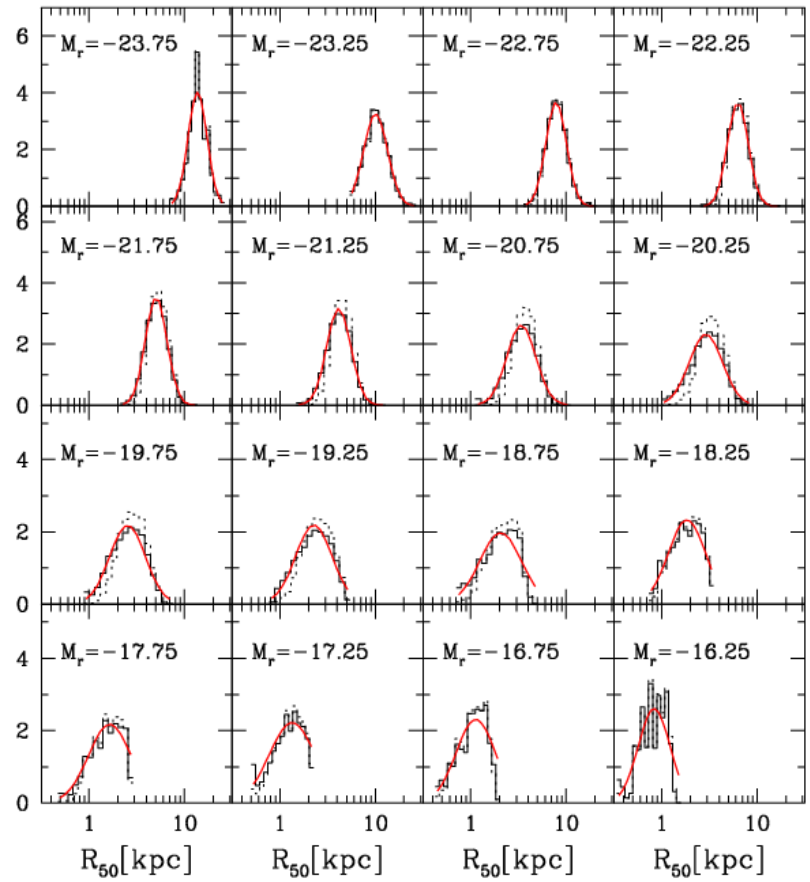
  ……

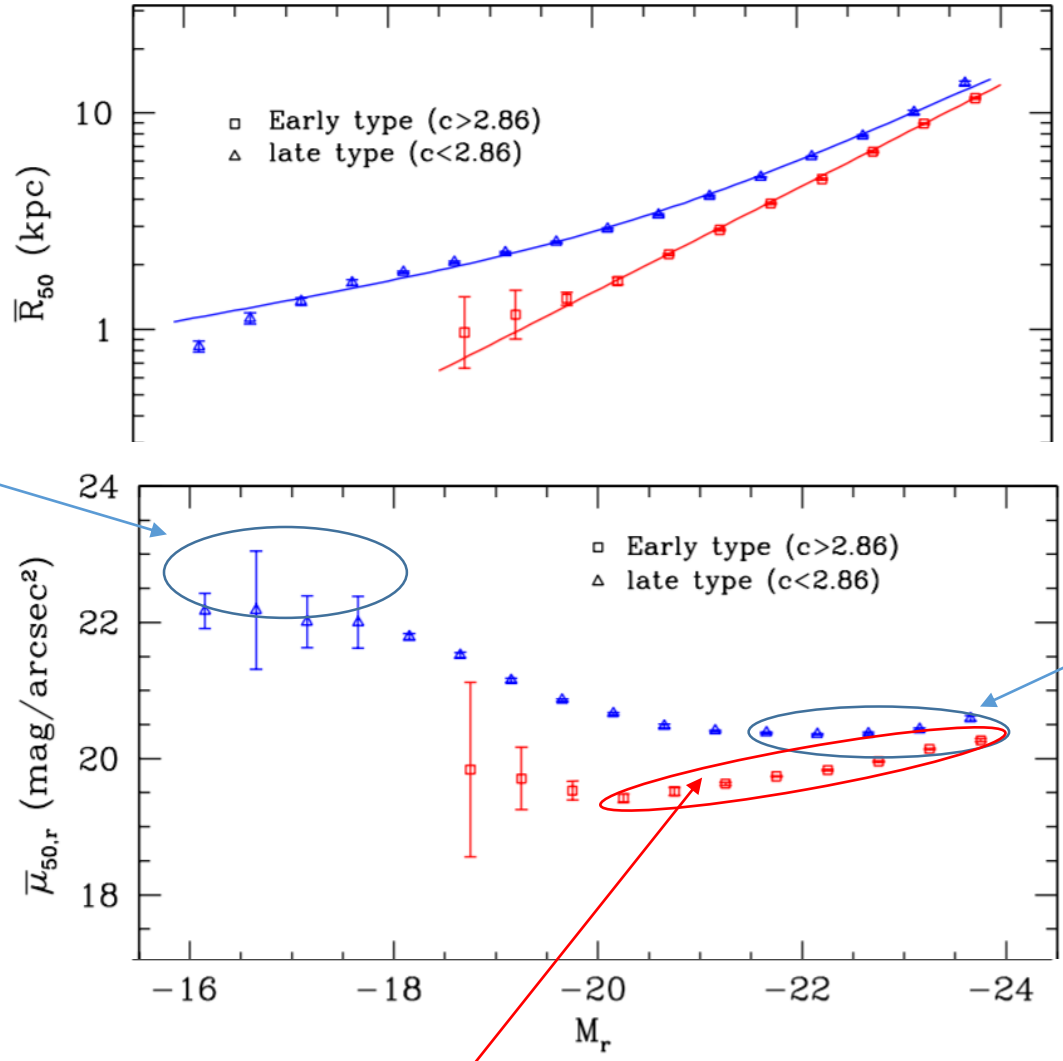
星系尺度的分布函数(Shen et al. 2003)

Early type c>2.86



Late type c<2.86



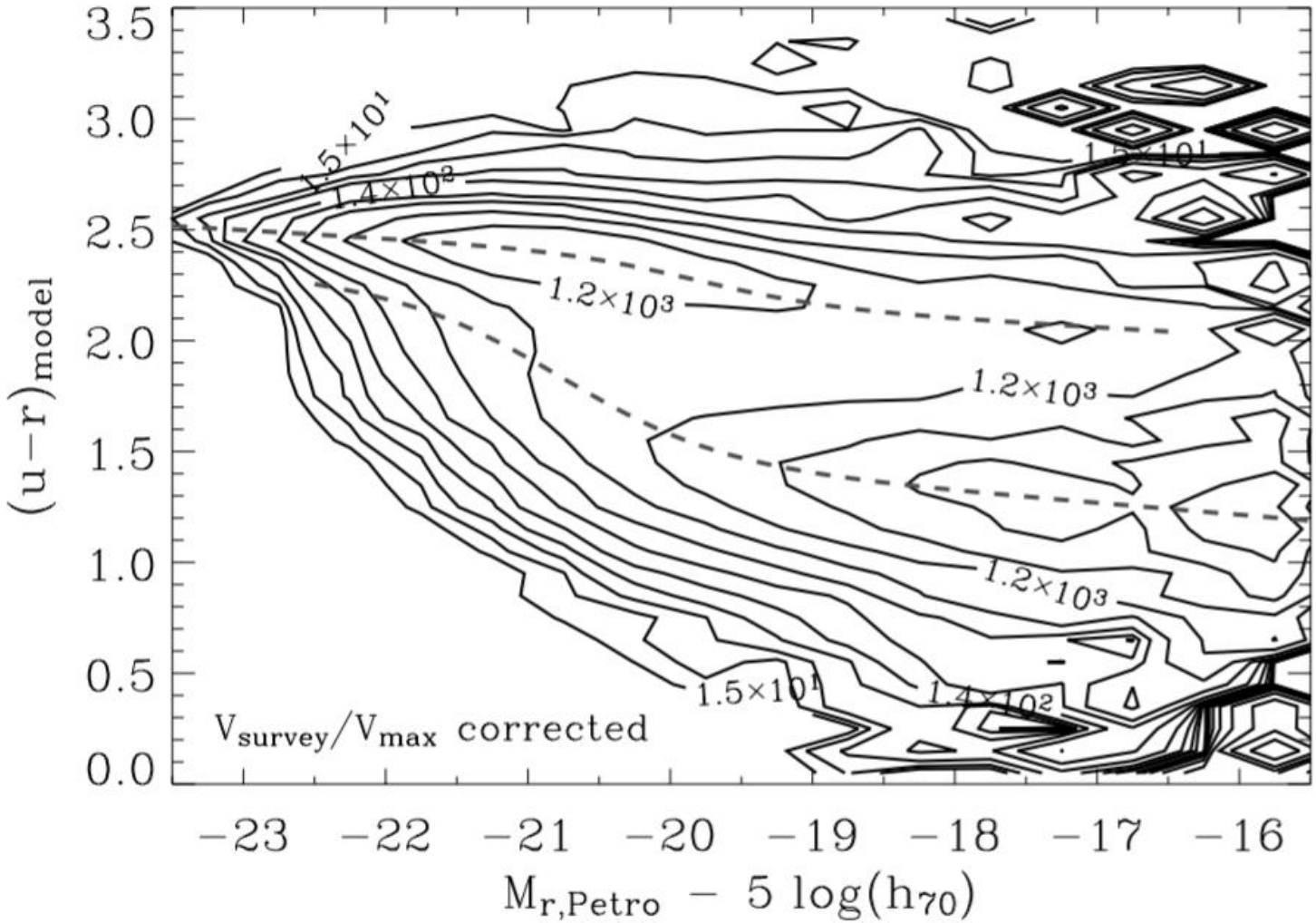


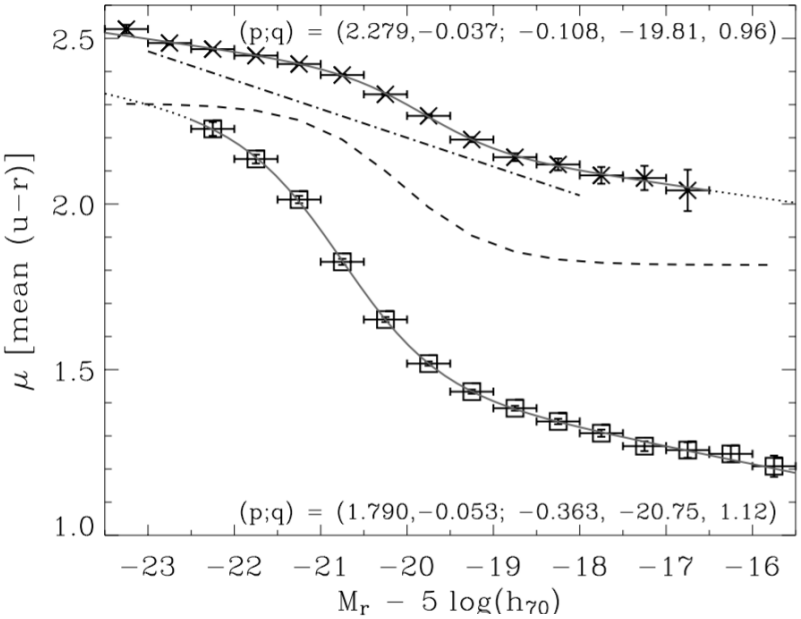
Low surface brightness galaxies

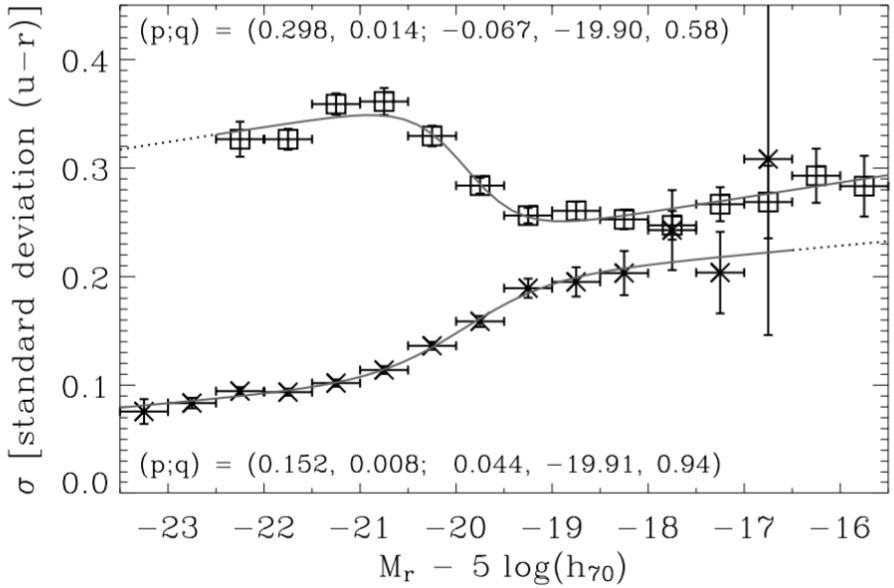
Freeman law 1970

Kormendy relation 1977

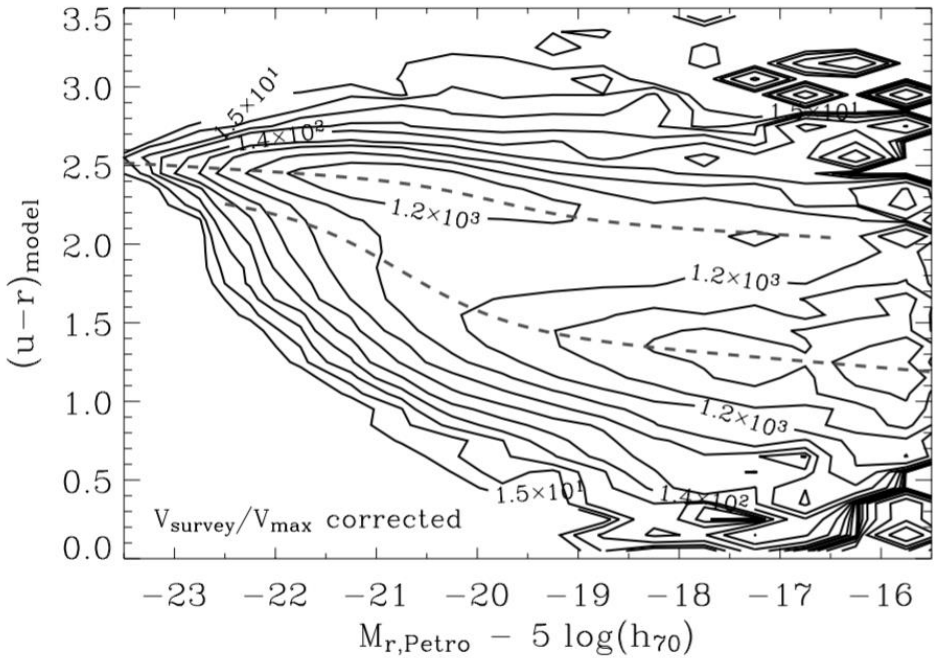
星等颜色分布(Baldry et al. 2004)







Baldry et al. 2004

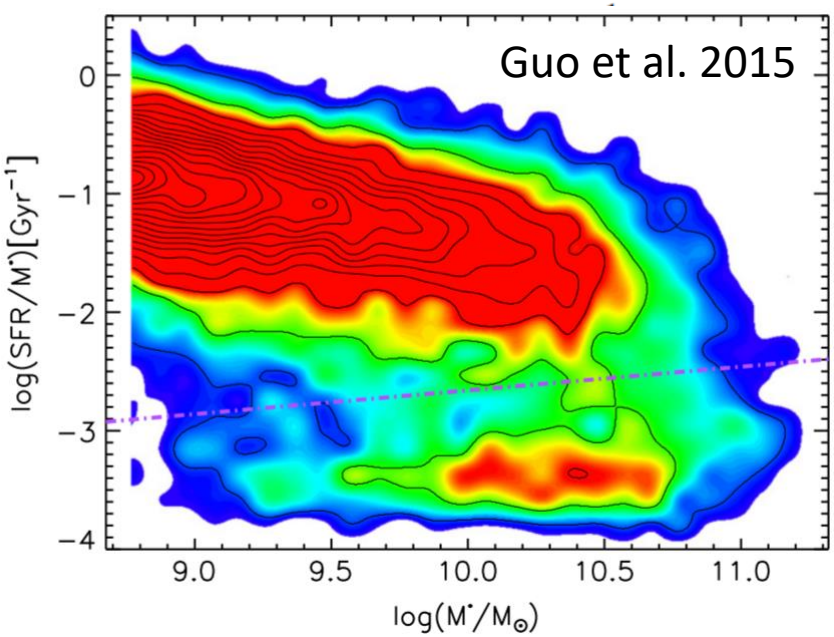


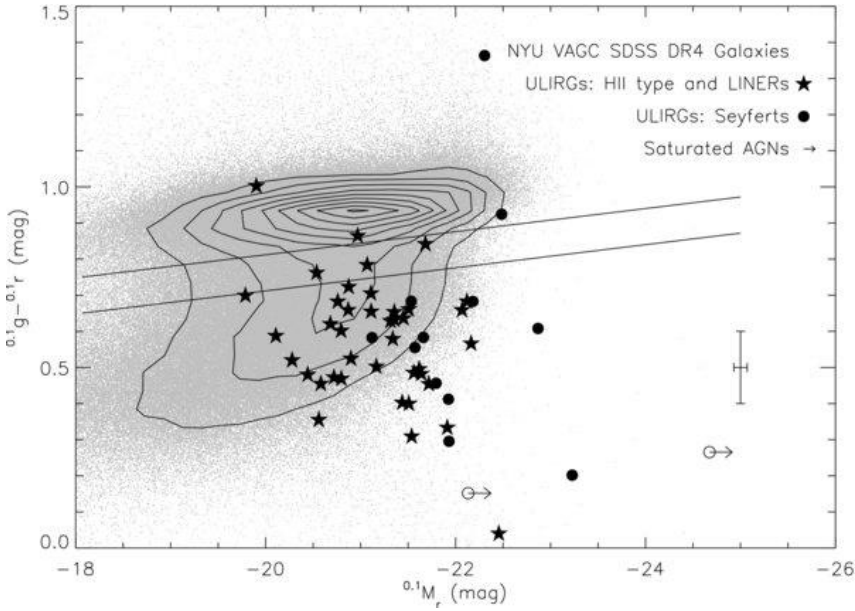
颜色-星等关系（标度关系？）

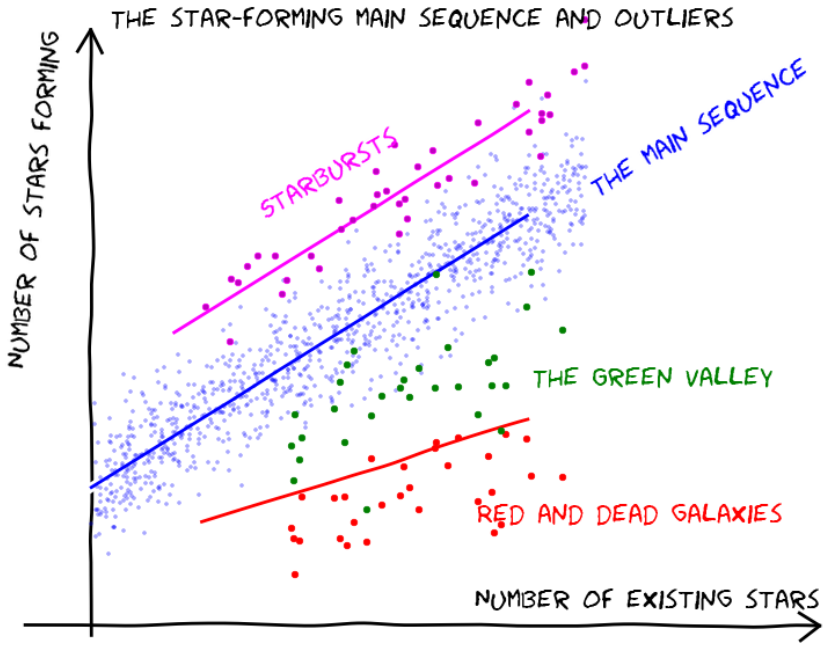
- sSFR-质量（光度）关系
- SFR-质量（光度）关系
  - Main sequence

$$\log(\mathrm{sSFR_t}) = \alpha \times \log\left(M^*/10^{10.5}\,M_\odot\right) + \beta$$

$$\alpha = -0.44 \text{ and } \beta = -1.68.$$



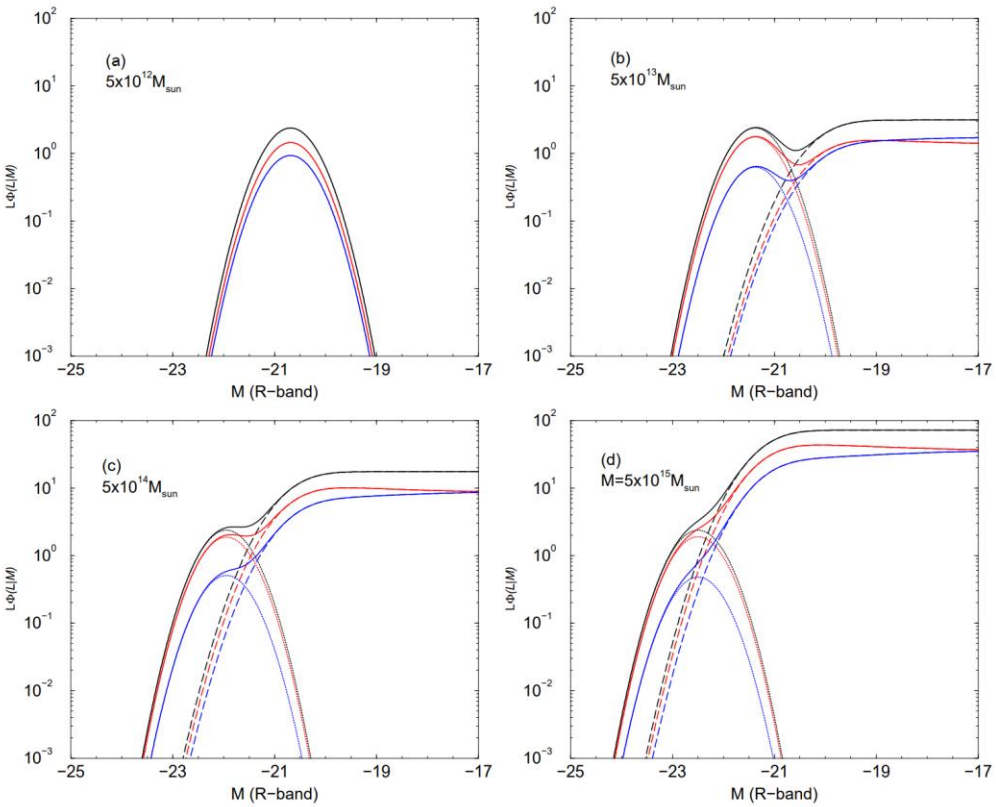




基于暗晕质量的Conditional luminosity function model（Yang et al., 2003, 2005)

- 暗晕→ 星系群
- 大星系+卫星星系
- 中央星系，卫星星系
- Halo Occupation Distribution (HOD)
- 条件分布函数和两点相关函数结合
  - 从统计上完全Mock星系的分布

Cooray 2005



（三）标度率（scaling relation）

- 定理：
  - F=ma
- 量纲
- 弥散
  - 观测误差
  - 内禀弥散

3.1 动力学标度关系

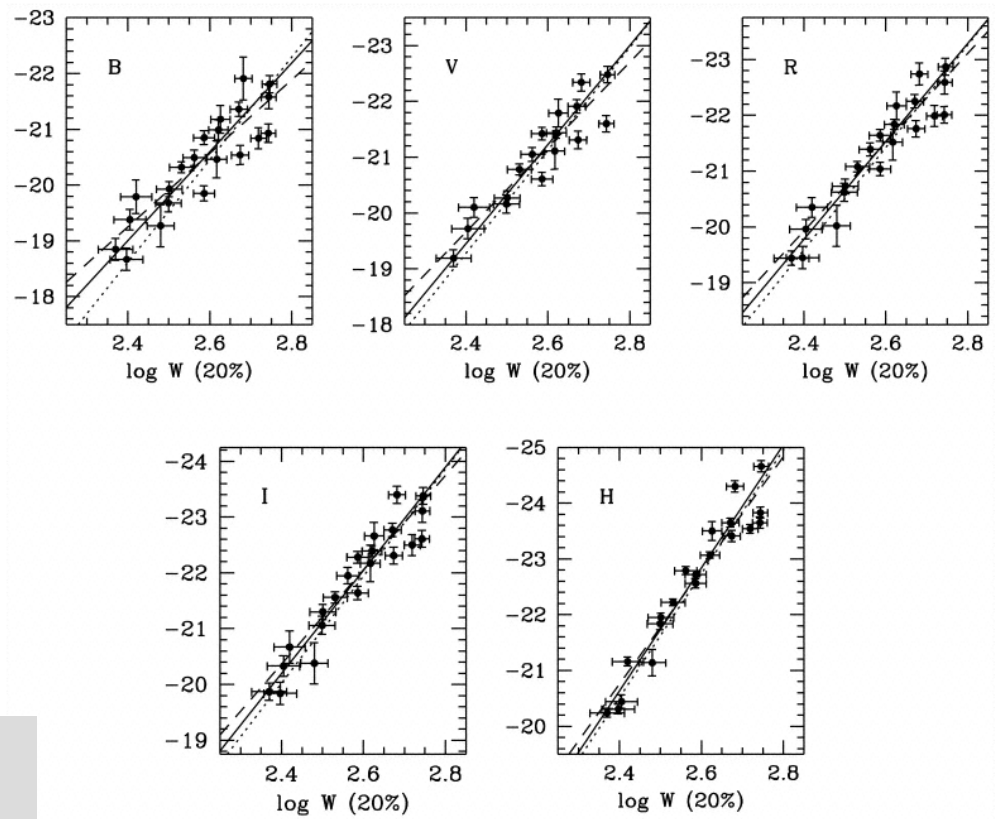
- 星系中各种组分在质量上的关系
  - Tully-Fisher关系
  - Faber-Jackson关系
  - Fundamental plane
  - M-sigma 关系

- 注意：大城市里面酒吧多，书店也多
  - 相关性 不等于 因果性

旋涡星系的 Tully-Fisher关系(1977)

- M=a log W +b
  - L=a*V$^{\alpha}$ （α≈4）
- 弥散：~0.2个星等
- W的测量：
  - 不依赖于距离
- 测定星系的距离

BVRIH$_{-0.5}$ Tully-Fisher relations for spiral galaxies with Cepheid distances, using 20% line width. Solid lines represent the bivariate fits, while the dotted and dashed lines represent inverse and direct fits, respectively.



$$L_H \approx 3\times10^{10}\left(\frac{V_{\max}}{196\ \mathrm{km/s}}\right)^{3.8} L_{H,\mathrm{solar}}$$

The "Physics" of Tully-Fisher

gravity: $V^2 = \frac{GM}{R} \Rightarrow M \sim RV^2$

mass-to-light ratio: $M = L\left(\frac{M}{L}\right)$

surface brightness: $\Sigma_{SB} = \frac{L}{\text{area}} \sim \frac{L}{R^2} \Rightarrow L \sim R^2 \Sigma$

so

$$M \sim M$$

$$RV^2 \sim L\left(\frac{M}{L}\right)$$

$$\sqrt{\frac{L}{\Sigma}}\, V^2 \sim L\left(\frac{M}{L}\right)$$

$$\Downarrow$$

$$L \sim \frac{V^4}{\Sigma (M/L)^2}$$

$$M_{halo} = \rho_{200} \cdot \frac{4}{3}\pi R_{200}^3$$

$$V_c^2 = \frac{G M_{halo}}{R_{200}}$$

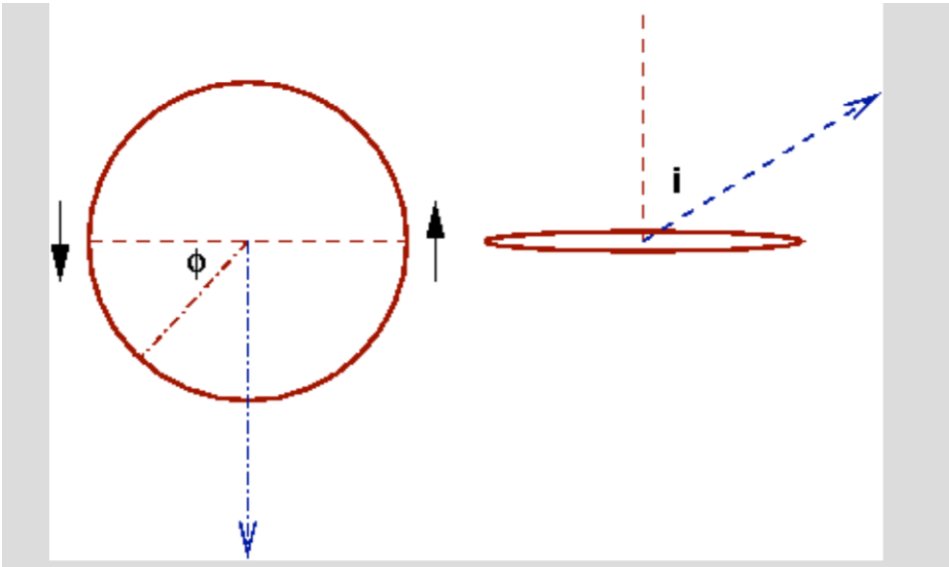
$$V_c \propto R_{200}$$

$$M_{halo} \propto V_c^3$$

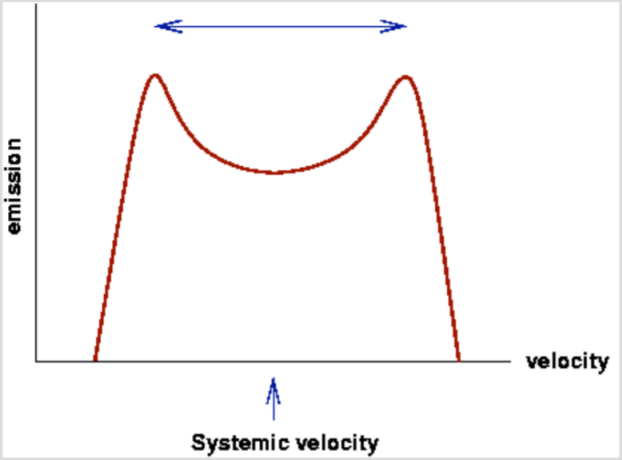
$$M^* \text{ —— } M_{halo}$$

旋转速度的测量

$$V_r(R,i) = V_{sys} + V(R)\sin i\cos\phi$$



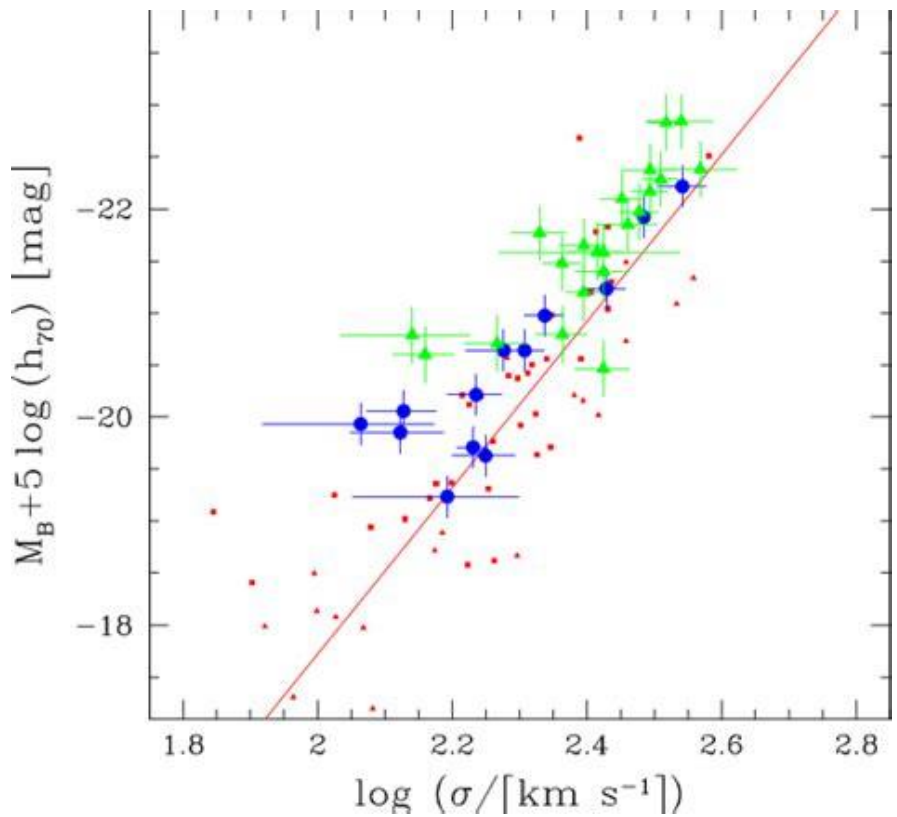
$$W \approx 2V_{\max}\sin i$$



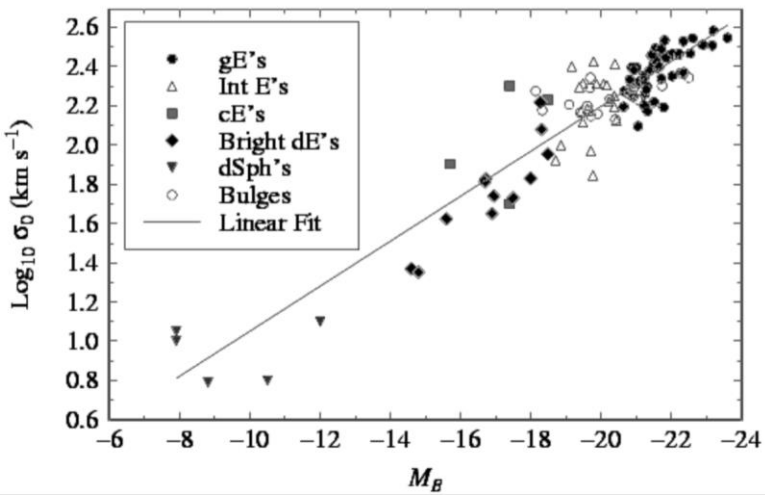
椭圆星系的Faber-Jackson关系 (1976)

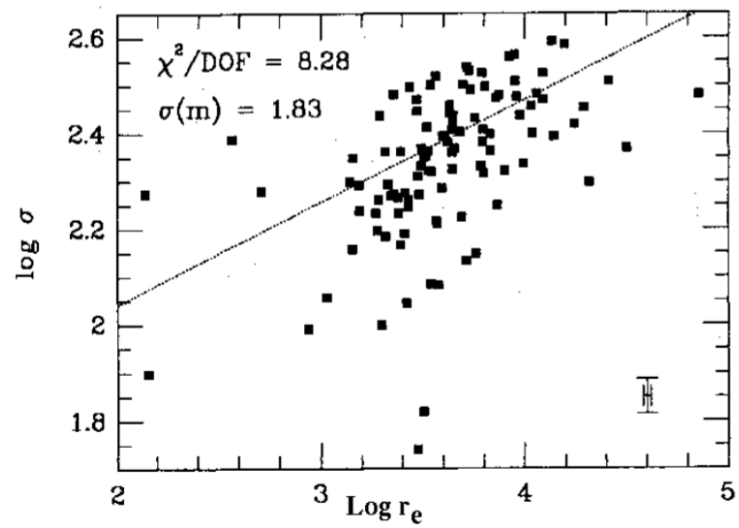
- L∝ σ~4
  - σ：恒星的速度弥散度
  - 关系的弥散~0.4mag

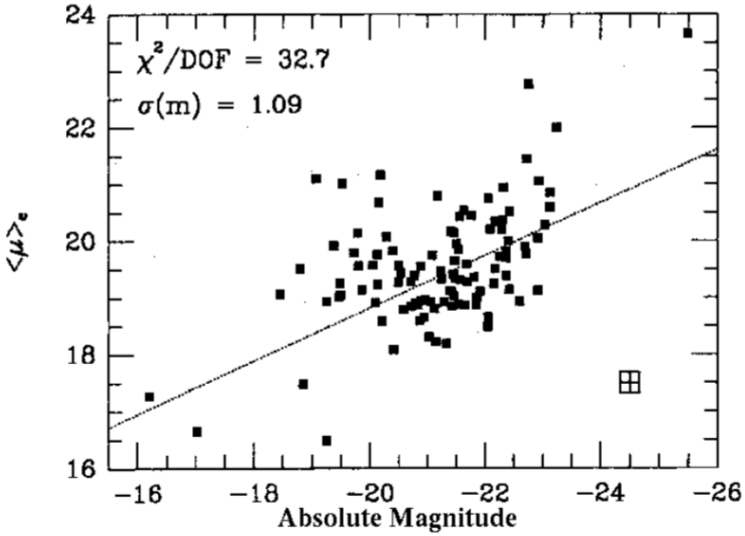
$$\sigma = \sqrt{\frac{\sum_{i=0}^{N}(v_i - \bar{v})^2}{N}}$$

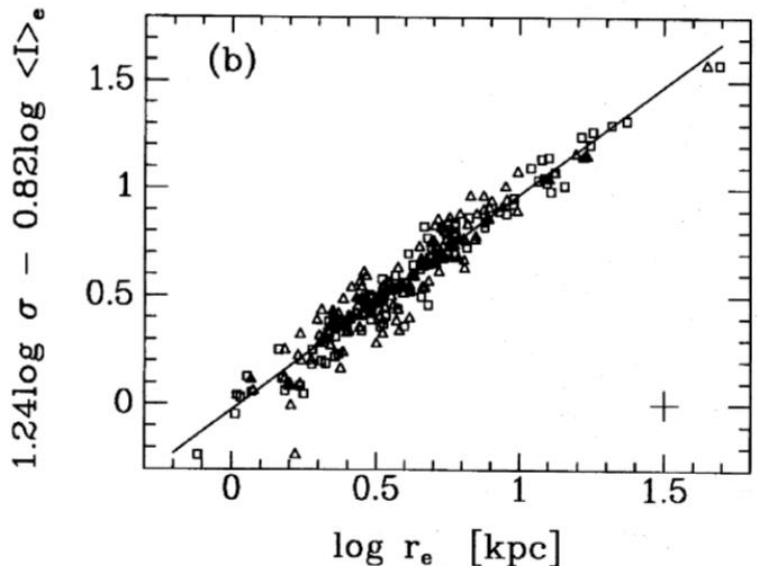


基本面（fundamental plane）









维里定理 $2K + U = 0$

$$U = -\frac{3}{5}\frac{GM^2}{R}$$ (半径为R，质量为M) 等密度

$$K = \frac{3}{2} M \sigma^2$$

$$\sigma^2 \propto G\frac{M}{R}$$

设 M/L 常数 $M \propto L$?

表面亮度 $\mu = L/\pi R^2$ μ：常数？

$$L \propto \frac{\sigma^4}{G^2 \mu}$$

基本面

$r_e \propto \sigma^{1.24} \langle I \rangle^{-0.82}$

- 可以测定距离
  - Ie， σ的测量不依赖于距离

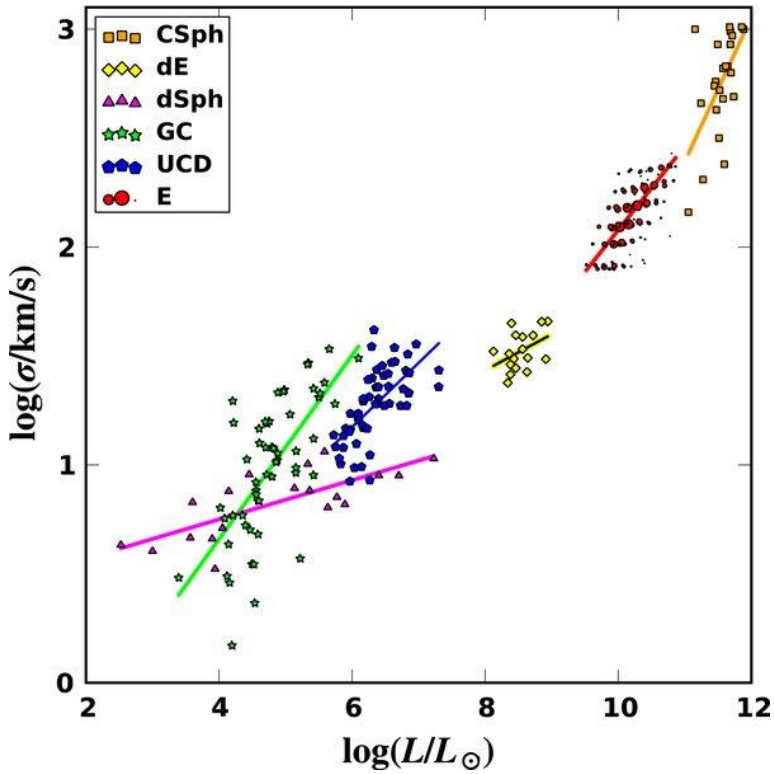
- 偏离维里面
  - 星系的质光比是常数吗？

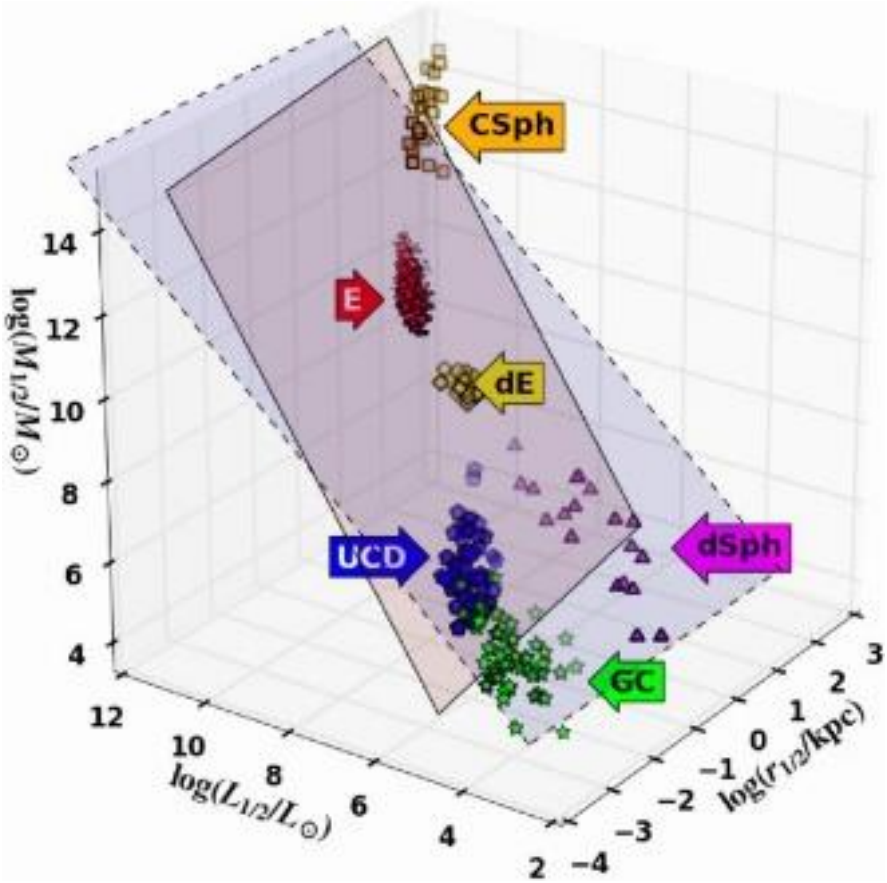
$$\sigma_r^2 = \frac{1}{3}\gamma\frac{GM_{\rm dyn}}{\frac{4}{3}R_{\rm h}} = \frac{1}{2}G\gamma\frac{\frac{1}{2}L}{R_{\rm h}^2}\frac{M_{\rm dyn}}{L}R_{\rm h} \propto IYR_{\rm h} \quad . \tag{3}$$

其中，Y 为系统的质光比，$Y = M_{\rm dyn}/L$，L 为系统的光度；I 为半质量半径处对应的平均面亮度，$I \propto \frac{1}{2}L/R_{\rm h}^2$。质光比可以分解成重子物质比例的倒数 ($M_{\rm dyn}/M_*$，$M_*$ 为系统的恒星质量) 与恒星质光比 (M_*/L) 的乘积：$M_{\rm dyn}/L = M_{\rm dyn}/M_* \times M_*/L$。若质光比 Y 为常数，则式 (3) 可以表示为：

$$\sigma^2 \propto IR_{\rm h} \quad . \tag{4}$$

其它球状动力学系统?



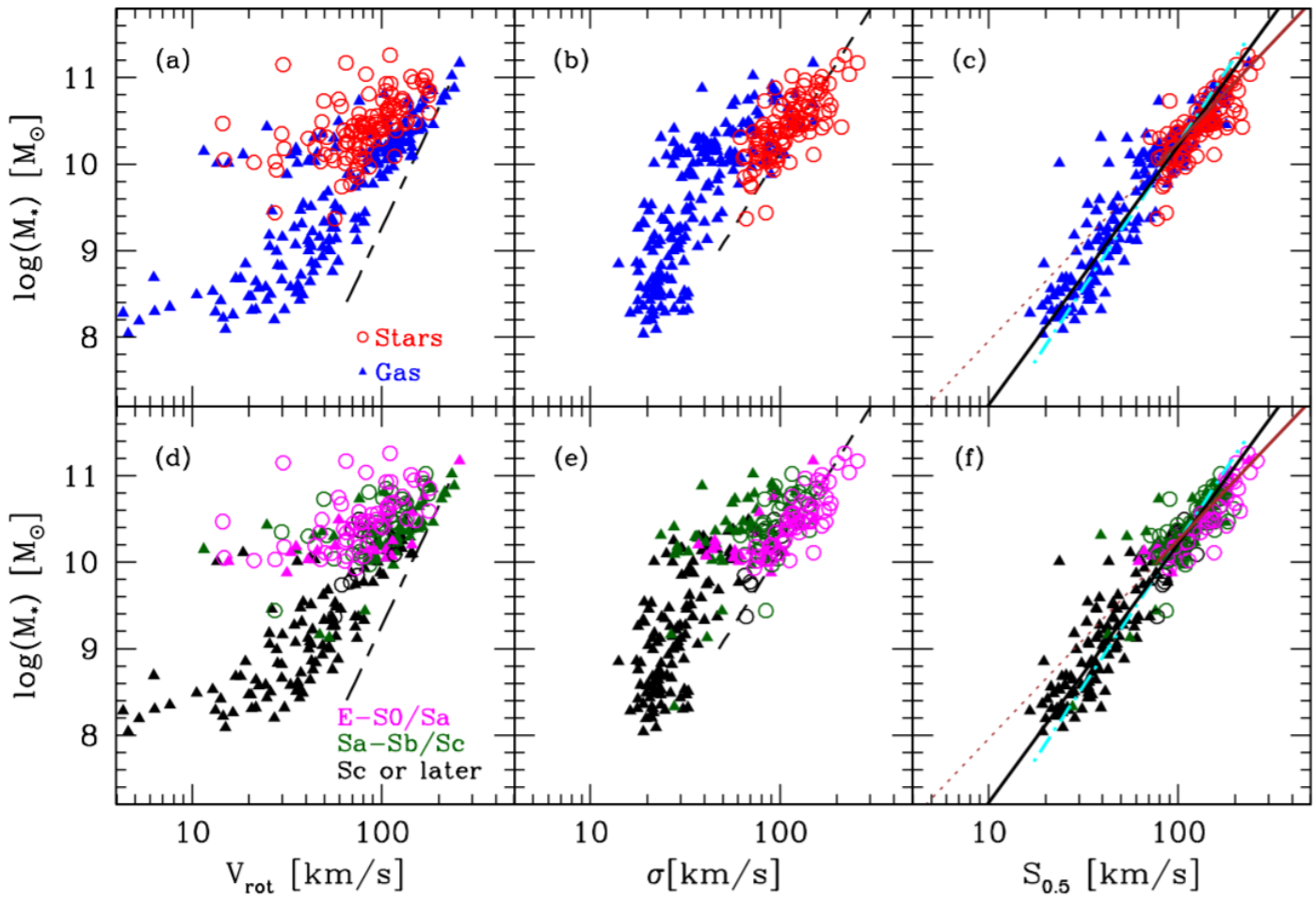


Tollerud et al., 2010, ApJ, 726, 2

Dynamical scaling relation

arXiv1410.3931

$$S_{0.5} = \sqrt{0.5V_{rot}^2 + \sigma^2}$$



黑洞的M-sigma关系

$$\frac{M}{10^8 M_\odot} \approx 1.9\left(\frac{\sigma}{200\ \mathrm{km\ s^{-1}}}\right)^{5.1}.$$

黑洞质量正比于核球质量

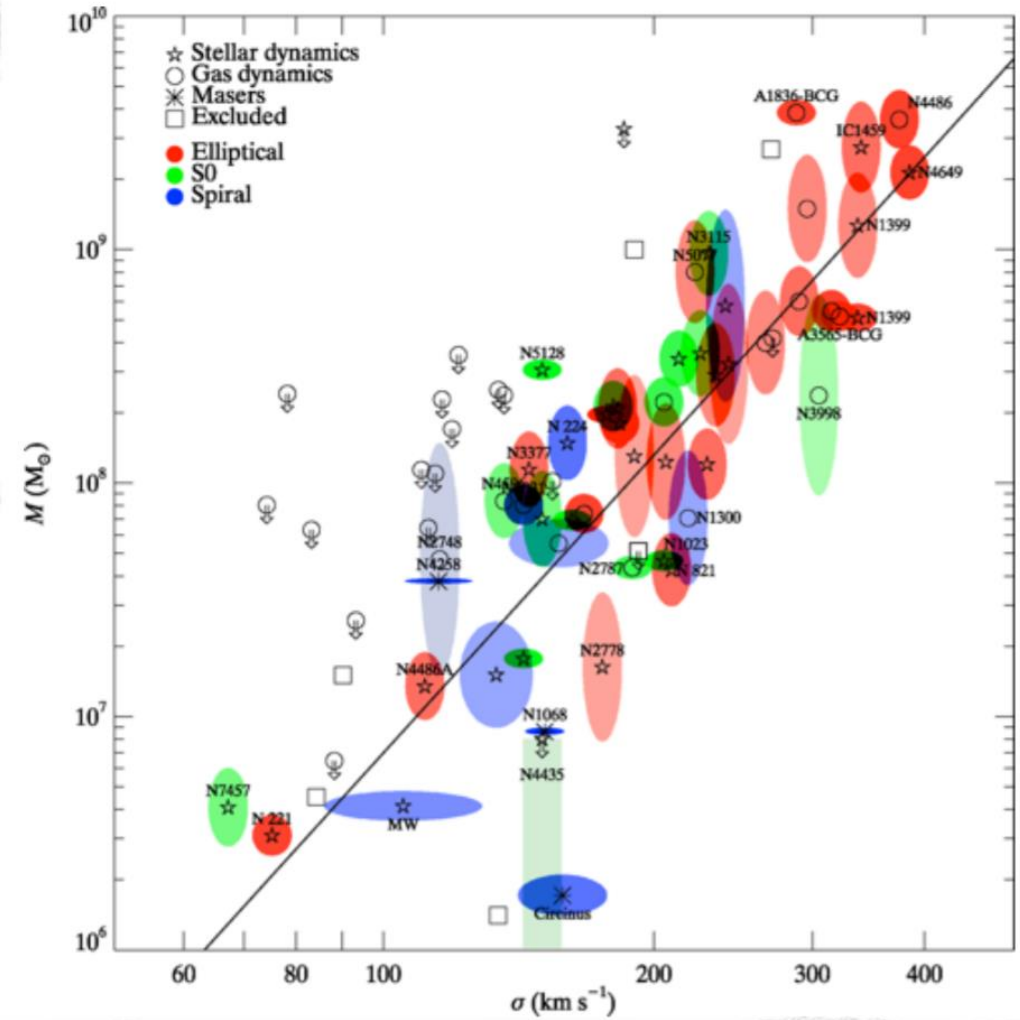
Mgal ~ 1000 MBH

黑洞和星系的共同演化

黑洞：~pc尺度

核球：Kpc尺度

黑洞质量怎么测量?



作业

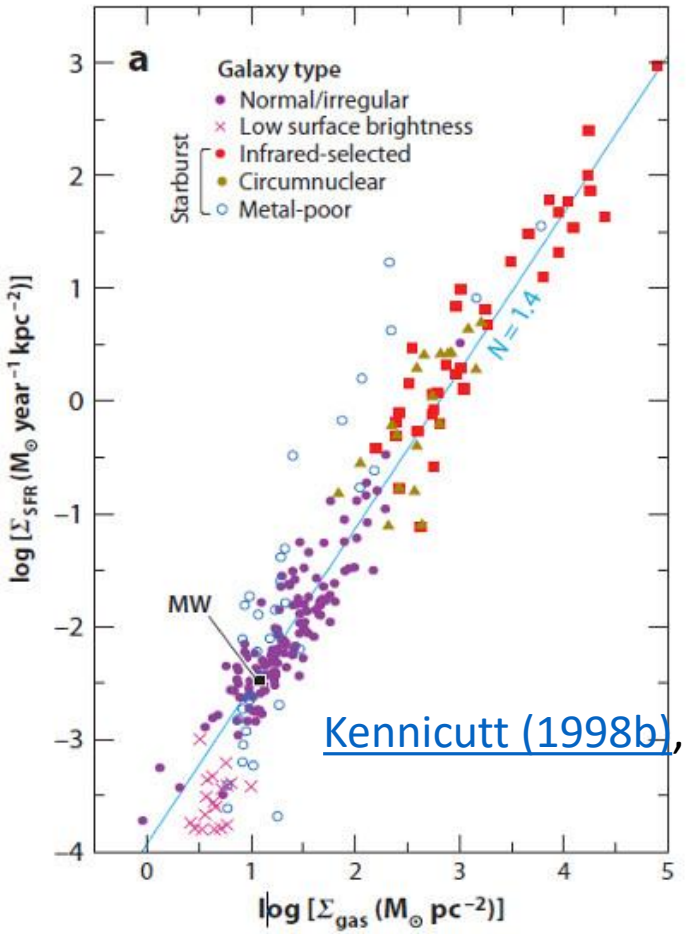
- 某旋涡星系和椭圆星系，其观测到的I波段的视星等都是18等；半光度半径内的表面亮度都是20mag/arcsec^2；其中旋涡星系观测得到的HI的速度展宽是200km/s，椭率为0.5，椭圆星系观测到的中心速度弥散度是200km/s，请问这两个星系的距离分别是多少？
  - 旋涡星系的 Tully-Fisher关系以及椭圆星系的Fundamental Plane的内秉弥散在0.2个星等左右，请问以上距离计算的误差是多少？

3.2 恒星形成相关

Kennicutt–Schmidt law

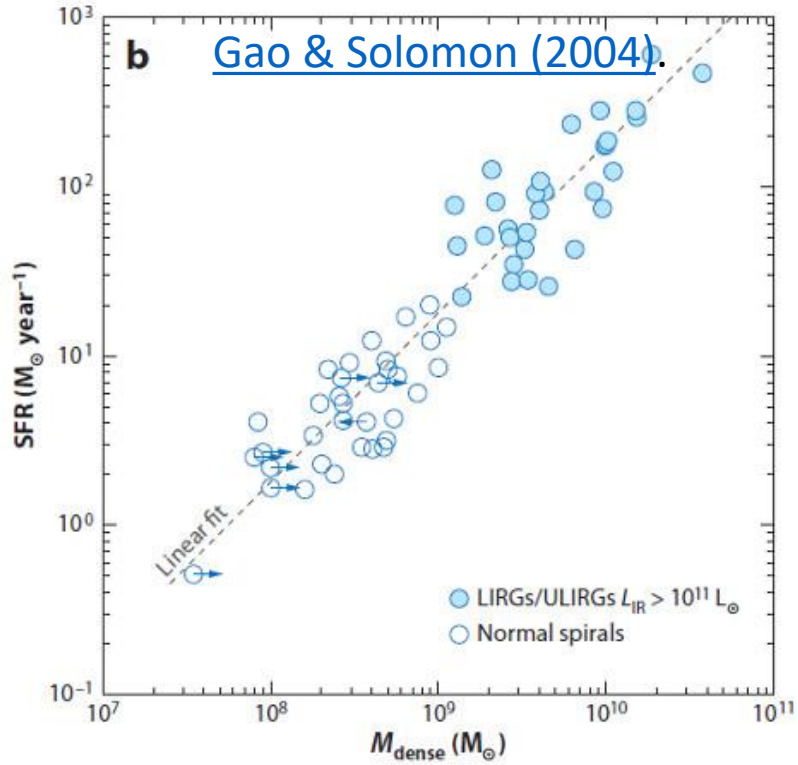
$$\Sigma_{SFR} = A\,\Sigma^{N}_{gas}$$

N=1.4



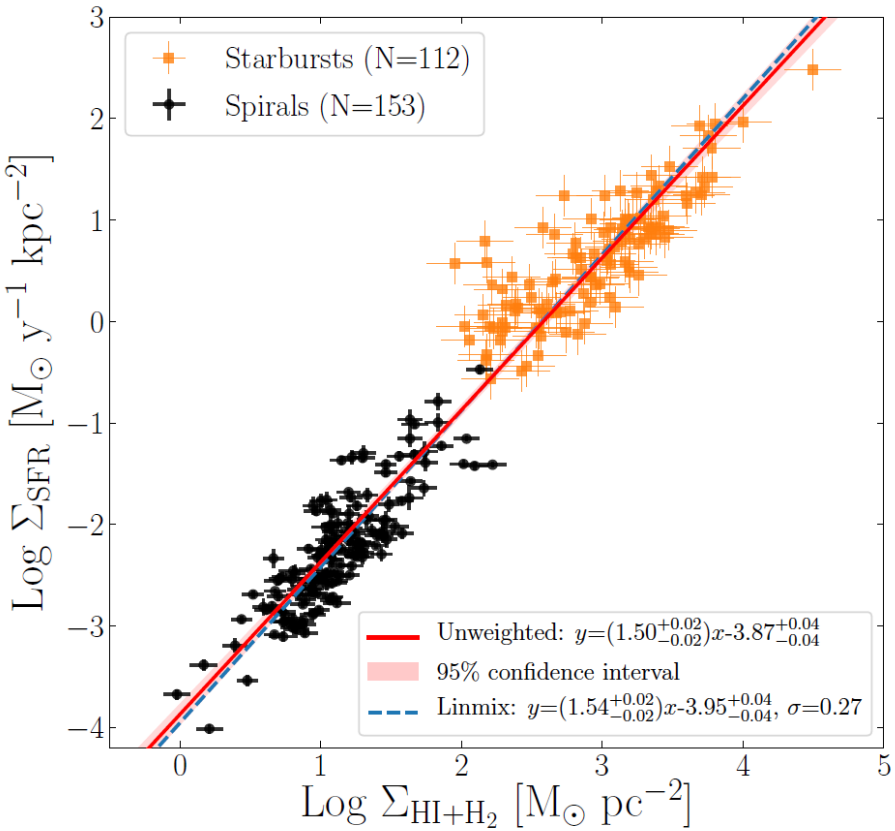
Kennicutt (1998b),

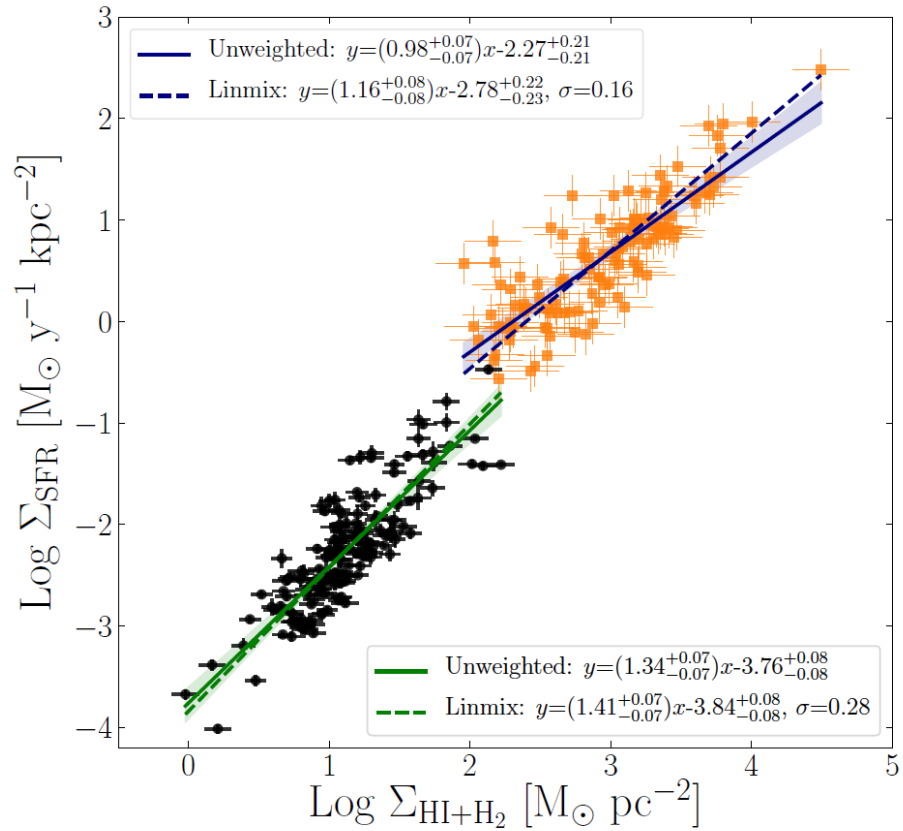
>Kpc尺度



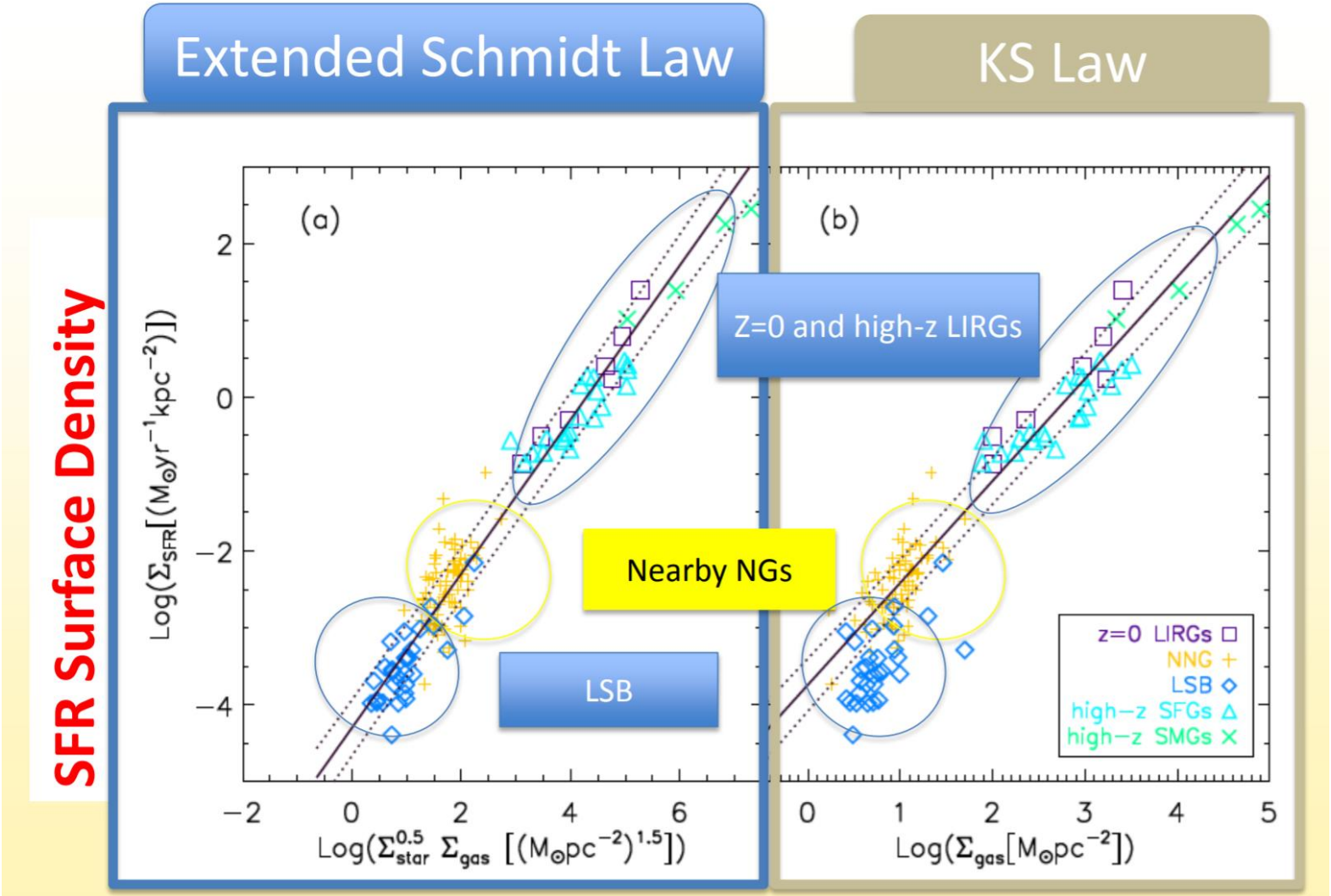
Gao & Solomon (2004).

Kennicutt & Reyes 2020 arXiv:2012.05363

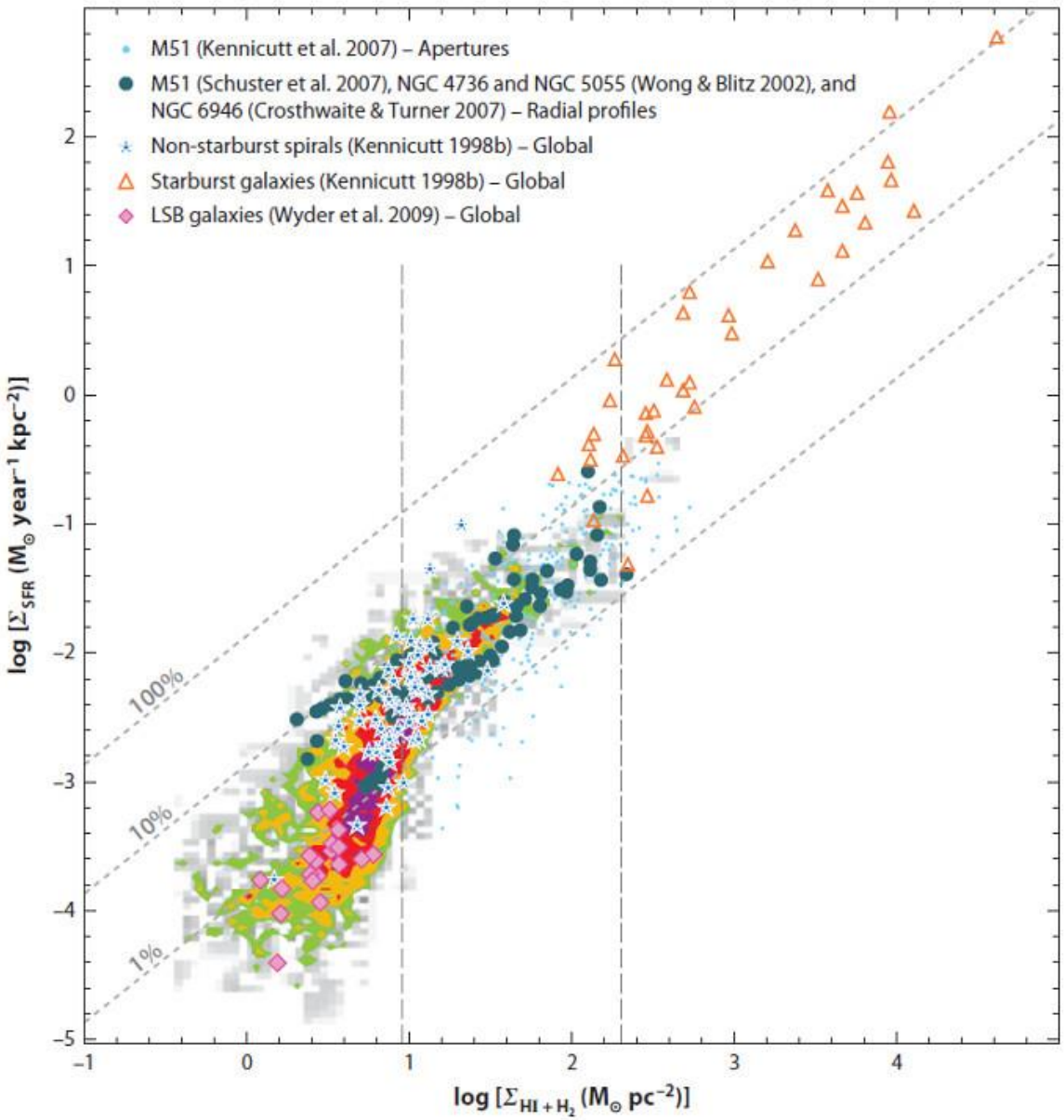


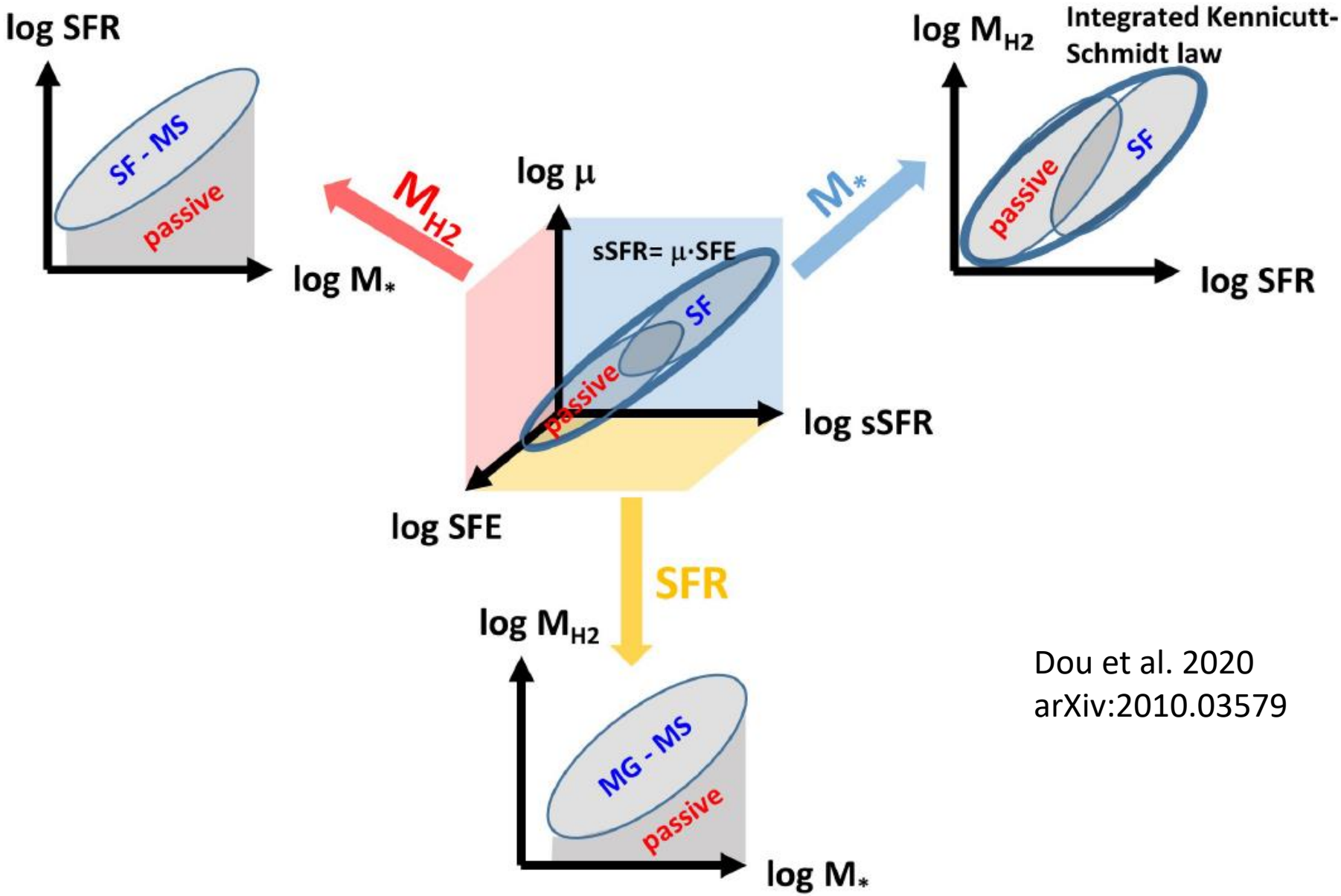


Extended Schmidt law （Shi et al. 2011）



Sub-kpc 尺度


M51 (Kennicutt et al. 2007) – Apertures
M51 (Schuster et al. 2007), NGC 4736 and NGC 5055 (Wong & Blitz 2002), and NGC 6946 (Crosthwaite & Turner 2007) – Radial profiles
Non-starburst spirals (Kennicutt 1998b) – Global
Starburst galaxies (Kennicutt 1998b) – Global
LSB galaxies (Wyder et al. 2009) – Global
100%
10%
1%
log [Σ_{SFR} ($M_\odot$ year^{-1} kpc^{-2})]
log [Σ_{HI+H_2} ($M_\odot$ pc^{-2})]
2
1
0
−1
−2
−3
−4
−5
−1
0
1
2
3
4

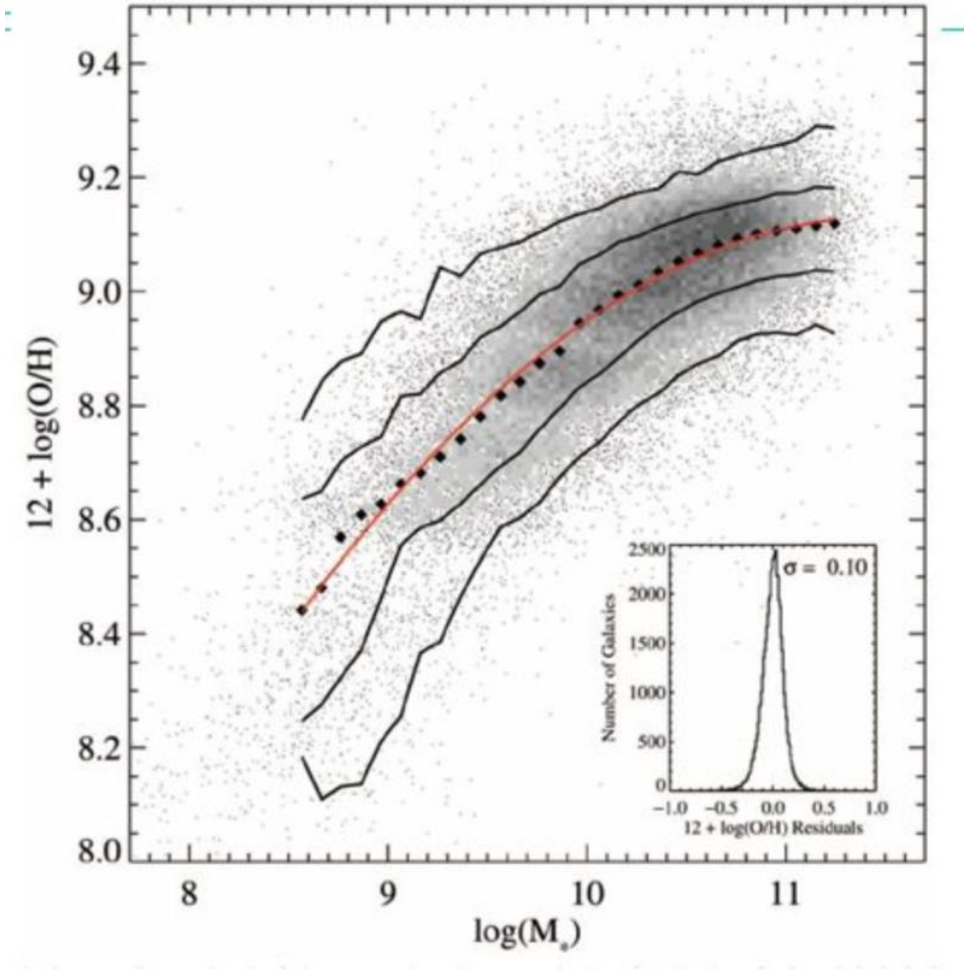


Dou et al. 2020
arXiv:2010.03579

3.3 金属丰度相关

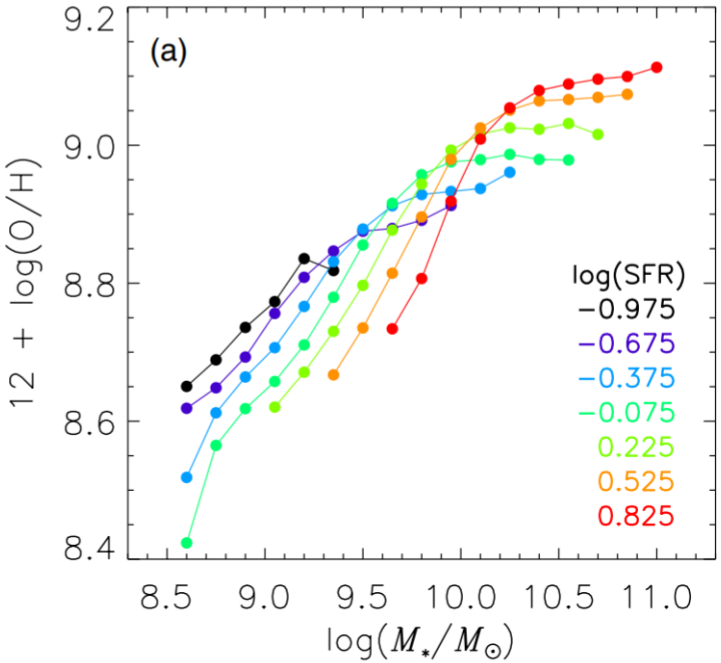
质量---金属丰度关系

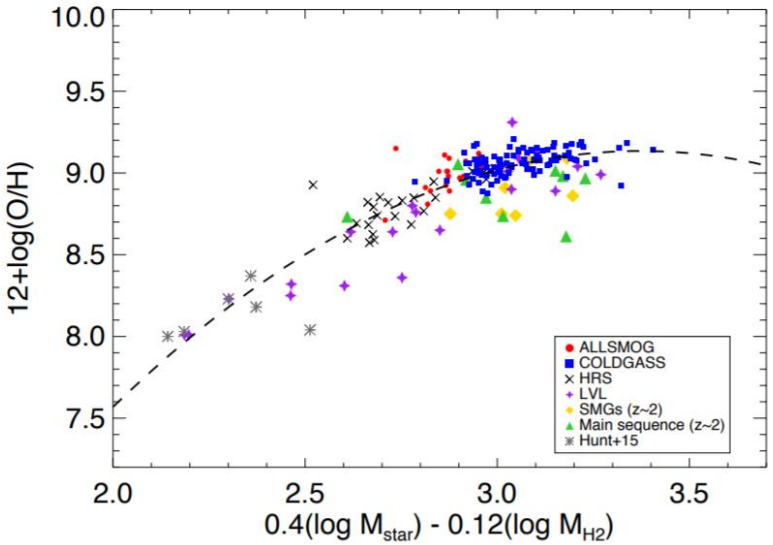
- 大质量星系
  - 势井更深
- 其它模型:
  - 大质量星系的恒星形成效率更



Fundamental metallicity relation (FMR)

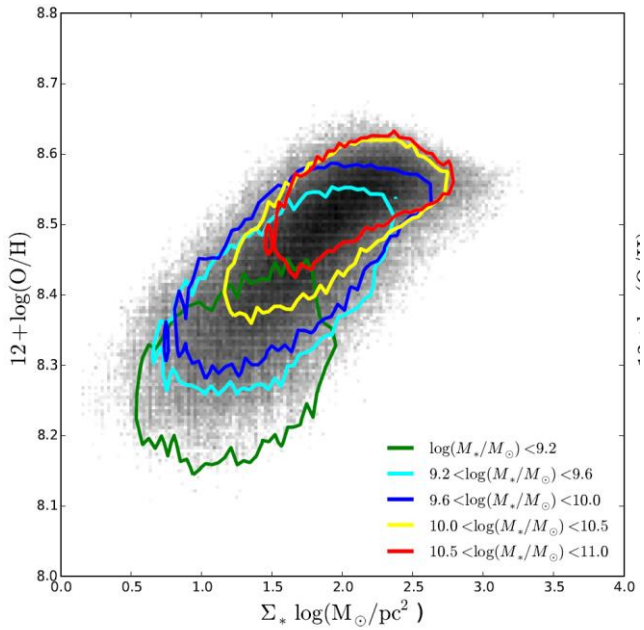
- M_*, gas-phase metallicity， SFR （Mannucci et al.2010）
  - 小质量端：气体内落稀释金属丰度
  - 大质量端：恒星形成星系富含气体

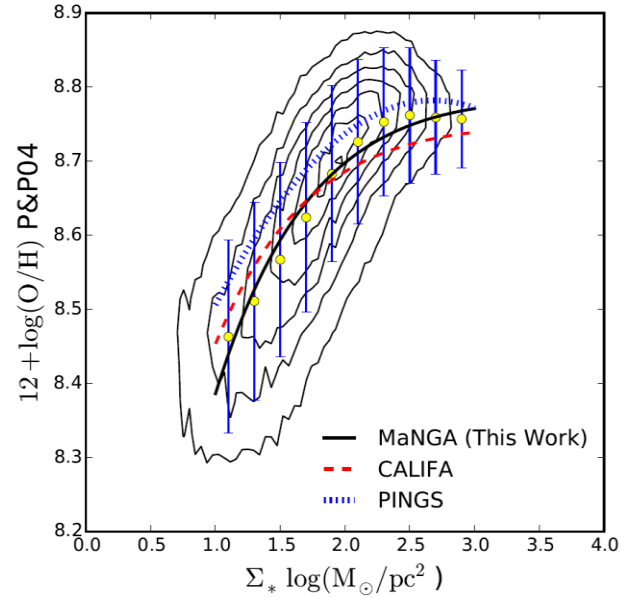




Local MZ relation: Surface mass density

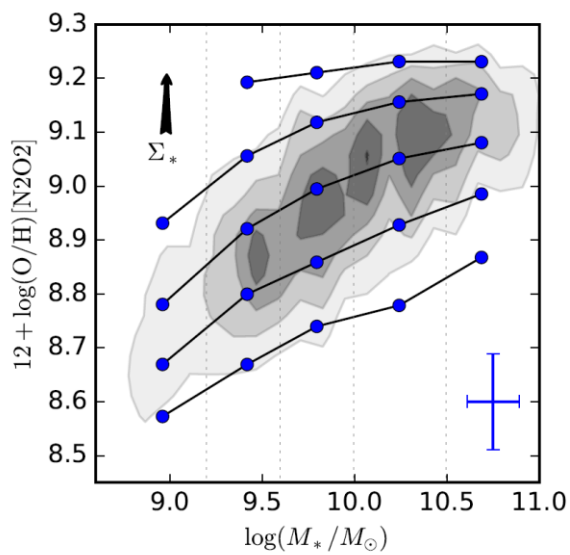
Barrera-Ballestero et al. 2016

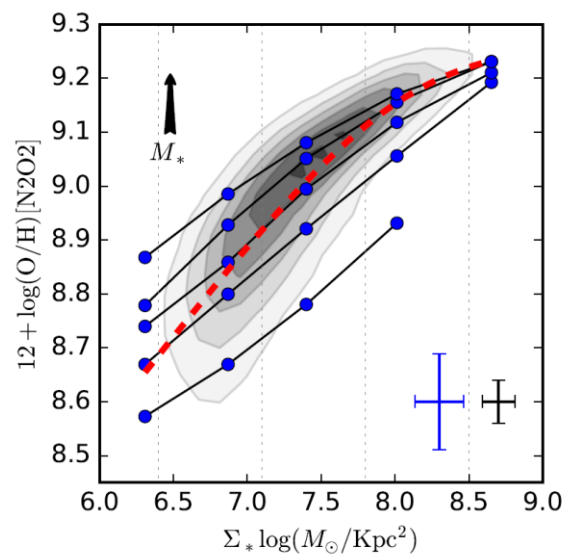




Gao et al. 2018 $(M_* - \Sigma_* - Z)$ relation

5





3.4 线性标度率的拟合

$$Y = a\,x + b$$

Famous linear scaling relations in astronomy

- period -luminosity relation of Cepheids
- M_{BH}-σ relation
- Tully-Fisher (L - V_{max}) relation
- Fundamental plane of ellipticals
- L-T, L-σ relation of groups and clusters
- All are statistical scaling relations, none of them are first principle like $F=ma$

Nature of the scaling relations

- Observables: (x_i, y_i) with error $(\Delta_{x,i}, \Delta_{yi})$
- First, we should find some correlations, e.g. rank analysis
- To the first order, all the correlations are linear
- $Y = a*X + b + \sigma$
  - σ is the intrinsic scatter, may not be a constant
- Observables maybe biased
  - e.g. some low-luminosity galaxies are not observed at given V_{max}
- Some observables may only be upper limits
  - E.g. we only get the upper-limit of L_x of some cluster

Ordinary Linear regression OLS(y|x)

- y_i with measurement error σ_i

$$\chi^2(a,b) = \sum_{i=1}^{N} \left(\frac{y_i - a - bx_i}{\sigma_i} \right)^2$$

Code: *fit* in numeric recipes

Error on both x and y

$$\chi^2(a,b) = \sum_{i=1}^{N} \frac{(y_i - a - bx_i)^2}{\sigma_{y\,i}^2 + b^2\sigma_{x\,i}^2}$$

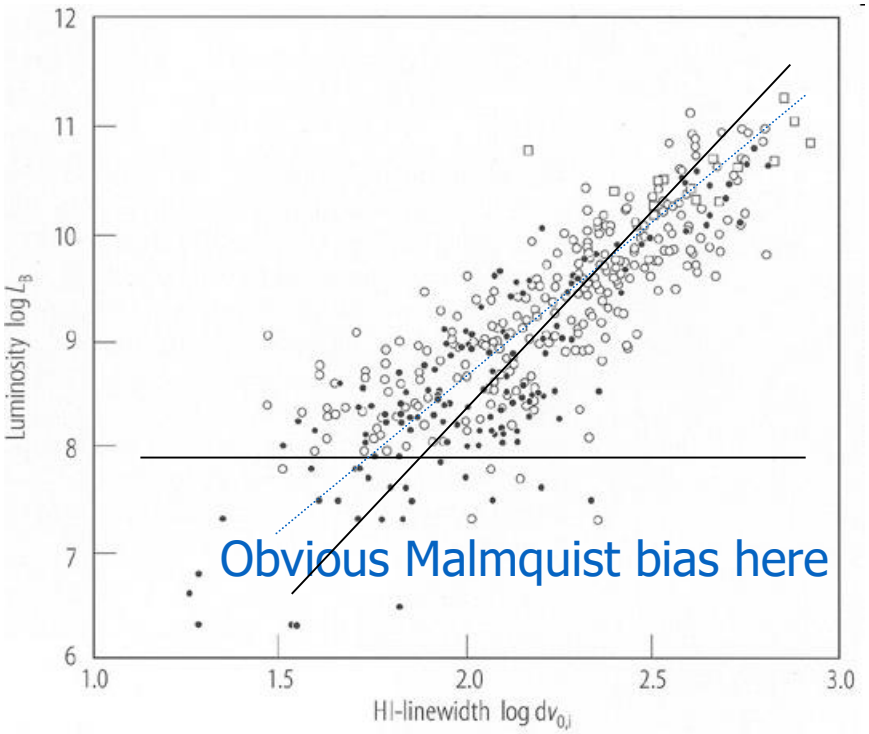
Code: *fitexy* in numeric recipes

b ~ biased to infinity

Caveat: choose proper parameterization

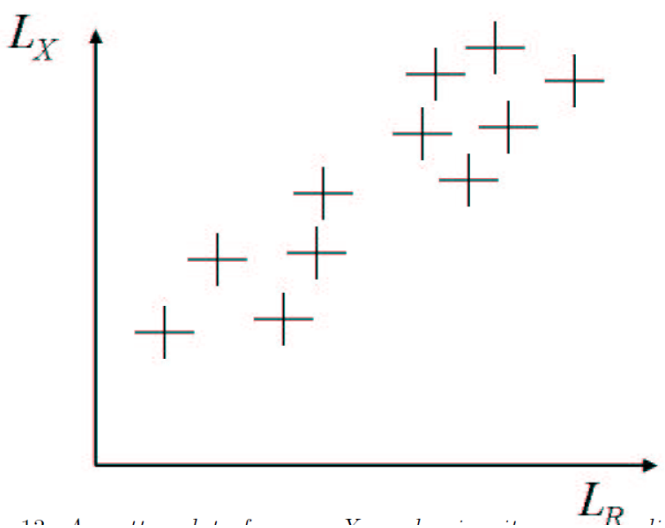
- If we fit M = *a* log W+ *b*, *a* will be biased to smaller values

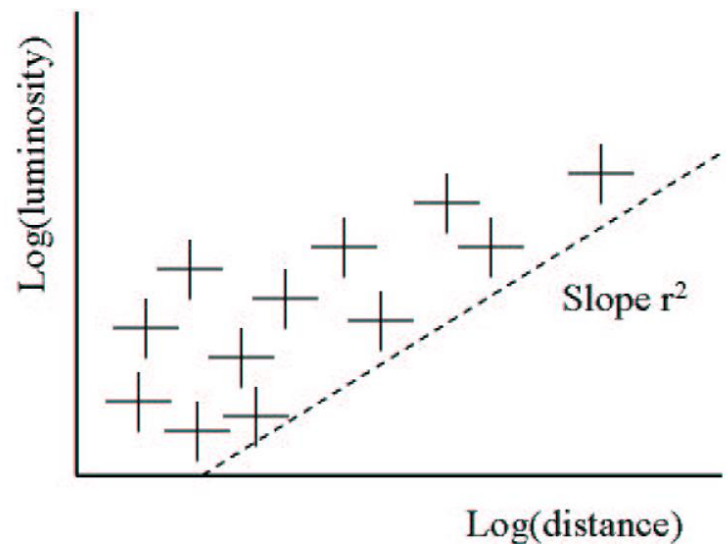
- Fit log W = a' M + b' is better
  - At given M, no obvious in W



Eddington(Malmquist) bias

- Distance dependent observable
  - Eddington (1915) Malmquist(1920)
  - In magnitude limit sample, more faint source scattered in than bright source scattered out





Six different linear regression

- Reference
  - Linear regression in astronomy I (1990, ApJ,364,104)
    - Different regression method
  - Linear regression in astronomy (1992ApJ...397...55)
    - Truncated, censored data
- IDL code: sixlin
  - Ordinary Least Squares (OLS) Y vs. X (c.f. linfit.pro)
  - Ordinary Least Squares X vs. Y
  - Ordinary Least Squares Bisector
  - Orthogonal Reduced Major Axis ;
  - Reduced Major-Axis
  - Mean ordinary Least Squares

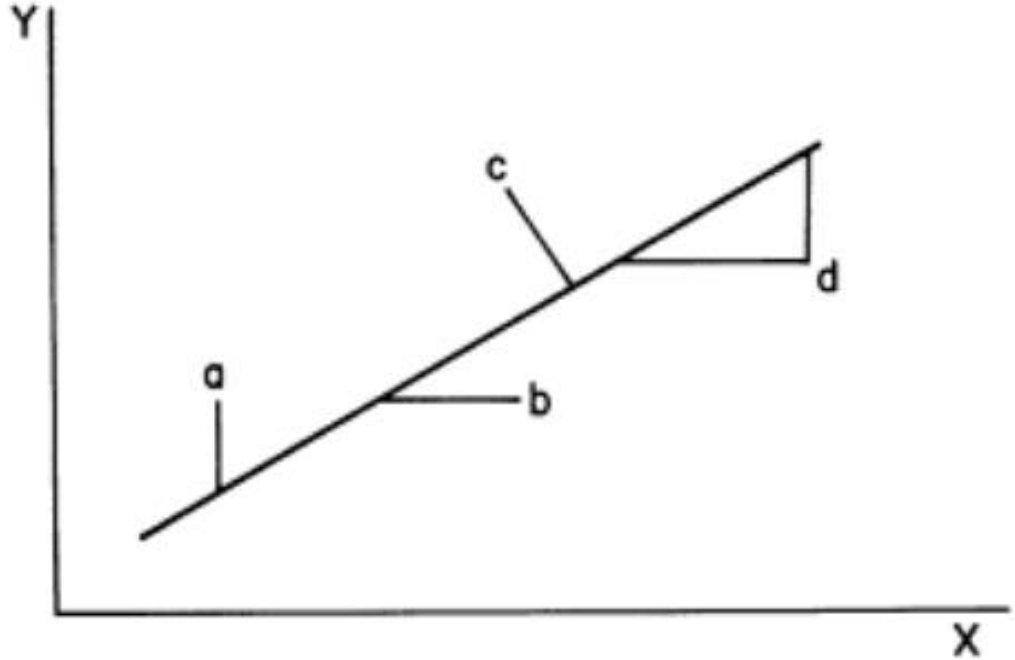


FIG. 1.—Illustration of the different methods for minimizing the distance of the data from a fitted line: (*a*) OLS($Y \mid X$), where the distance is measured vertically; (*b*) OLS($X \mid Y$), where the distance is taken horizontally; (*c*) OR, where the distance is measured vertically to the line; and (*d*) RMA, where the distances are measured both perpendicularly and horizontally. No illustration of the OLS bisector is drawn in this figure.

- The applicability of the procedures is dependent on the nature of the astronomical data under consideration and the scientific purpose of the regression.
- For problems needing symmetrical treatment of the variables, the OLS bisector performs significantly better than orthogonal or reduced major-axis regression.

Error on both x and y and with a constant intrinsic scatter σ

$$\ln L = -\frac{1}{2}\sum_i \ln\left(\sigma^2 + \sigma_{y,i}^2 + a^2\sigma_{x,i}^2\right) - \sum_i \frac{\left[\hat{y}_i - (a\hat{x}_i + b)\right]^2}{2(\sigma^2 + \sigma_{y,i}^2 + a^2\sigma_{x,i}^2)} + \text{constant.}$$

BCES (Akritas & Bershady, ApJ 470, 706 1996)

- Regression with correlated measurement errors and intrinsic scatter
  - allows for measurement errors on both variables
  - allows the measurement errors for the two variables to be dependent
  - allows the magnitudes of the measurement errors to depend on the measurements
- Intrinsic scatter: constant
- IDL code: BCES.pro (BCES: bivariate, correlate errors and scatter)

Special cases

Robust estimation

- Data with outlier

$$\text{minimize over } \mathbf{a} \qquad \sum_{i=1}^{N} \rho\left(\frac{y_i - y(x_i;\mathbf{a})}{\sigma_i}\right)$$

$$\sum_{i=1}^{N} |y_i - a - bx_i|$$

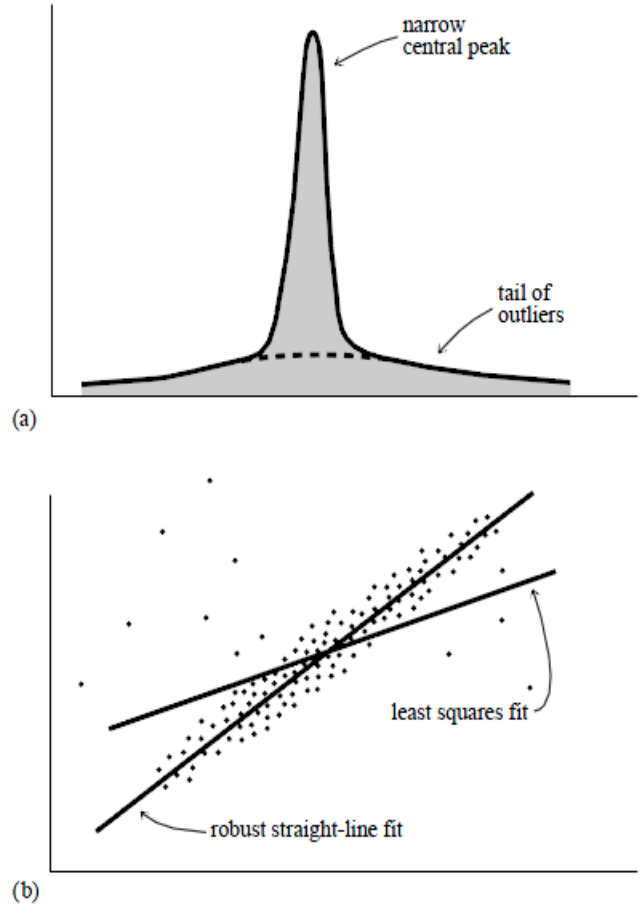
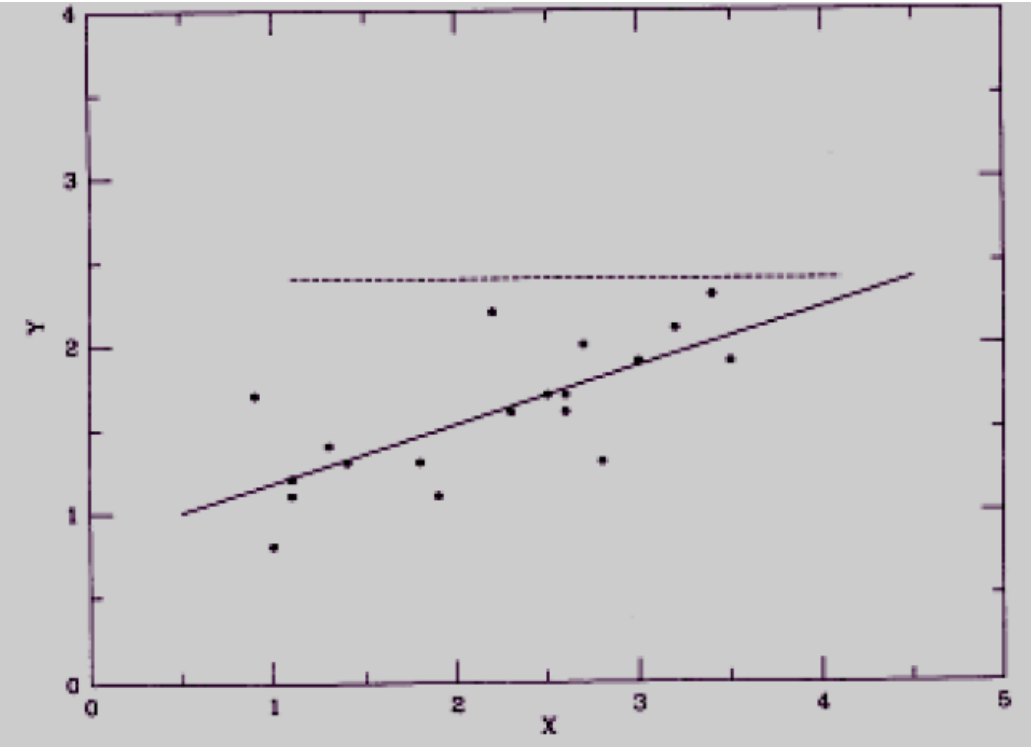


Figure 15.7.1. Examples where robust statistical methods are desirable: (a) A one-dimensional distribution with a tail of outliers; statistical fluctuations in these outliers can prevent accurate determination of the position of the central peak. (b) A distribution in two dimensions fitted to a straight line; non-robust techniques such as least-squares fitting can have undesired sensitivity to outlying points.

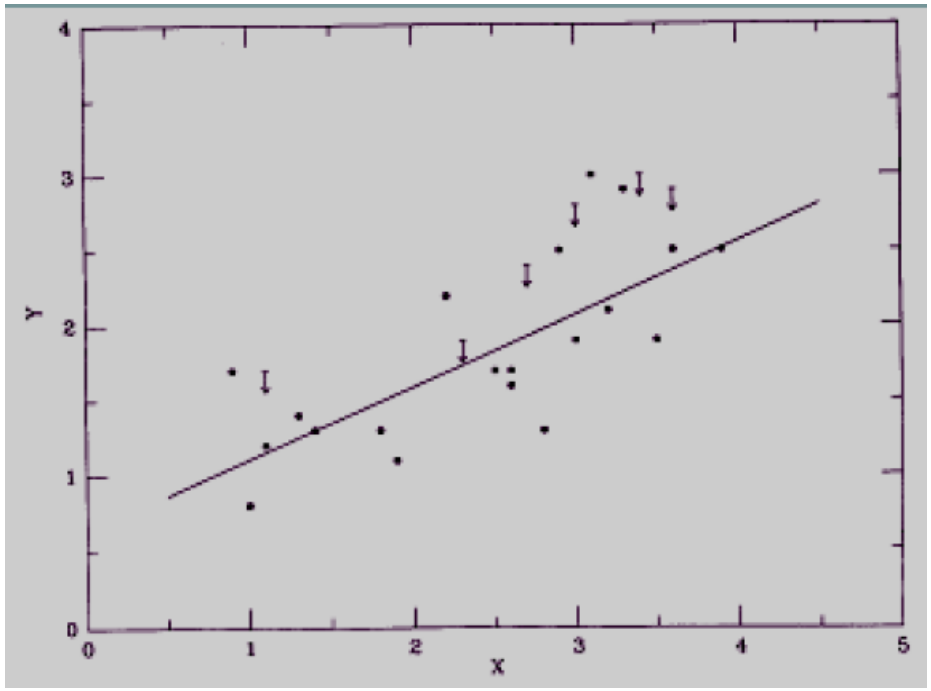
See Numeric recipes C15.7

Truncation due to flux limits



Malmquist bias in Hubble diagram (Deeming, Vistas Astr 1968, Segal, PNAS 1975)

Censoring due to non-detections

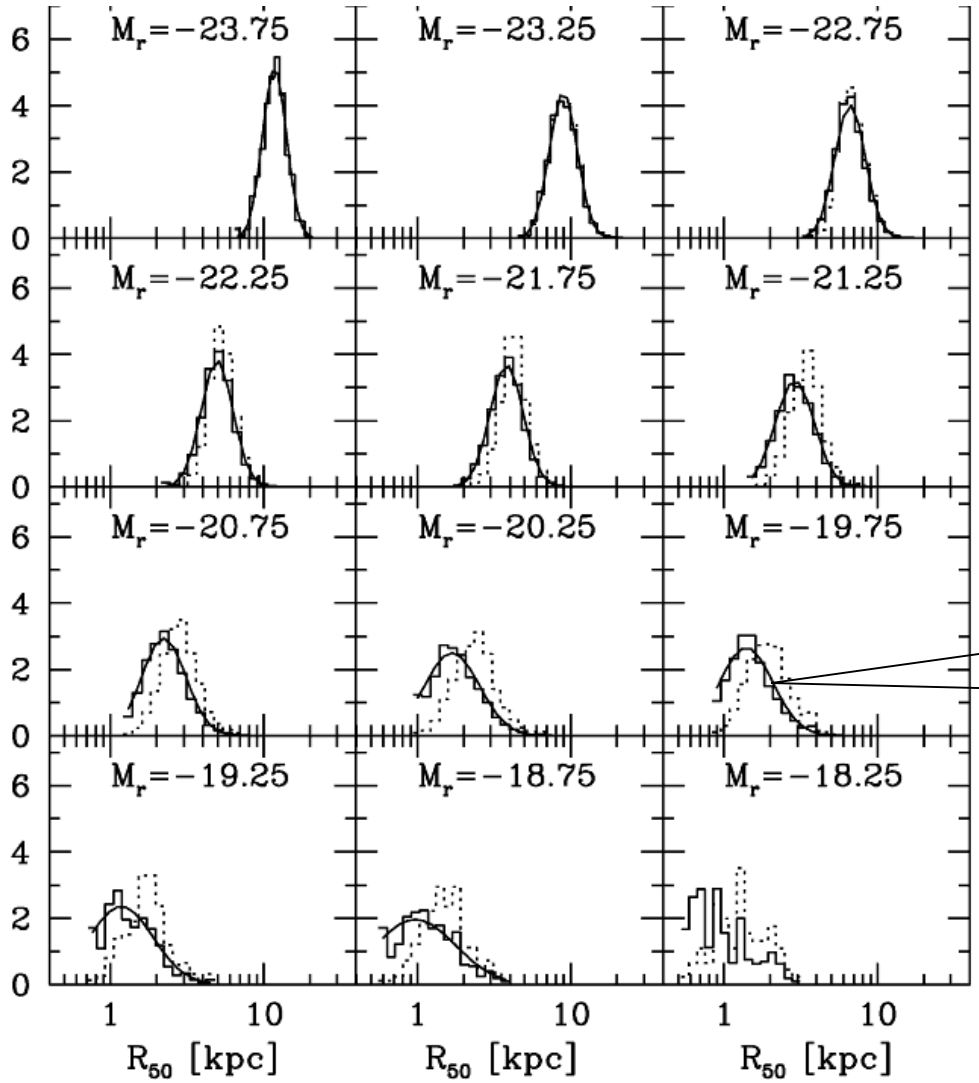


Presented for astronomy by Isobe, Feigelson & Nelson (ApJ 1986)
Implemented in Astronomy Survival Analysis (ASURV) package

A more straight-forward way

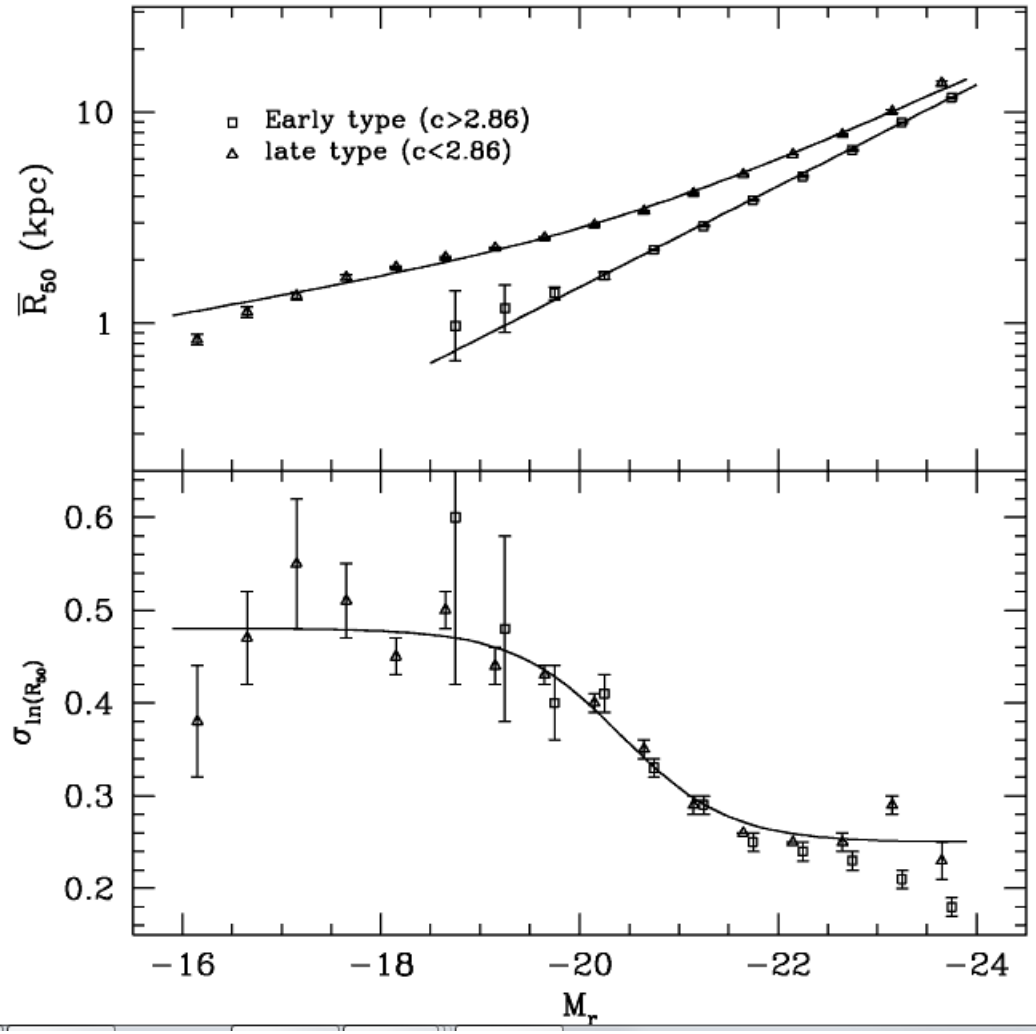
- Especially when amount of data is large in modern surveys
- First, at given bin of x, what is the distribution of y after correction for selection bias?
  - Is y Gaussian distributed? What is the scatter compared with its measurement error?
- Then what is the PDF(y|x) changes as function of x
  - Is this relation linear or non-linear?
- Build the likelihood function and fit the model parameters

L – R relation of galaxies (Shen et al. 2003)



We find, after correction for selection effect, at given Mr, Log *R* is intrinsically Gaussian distributed.

Data is biased here



We plot P(R|M) as function of M.

Intrinsic scatter is not a constant