

Fitting isochrones to open cluster photometric data

A new global optimization tool

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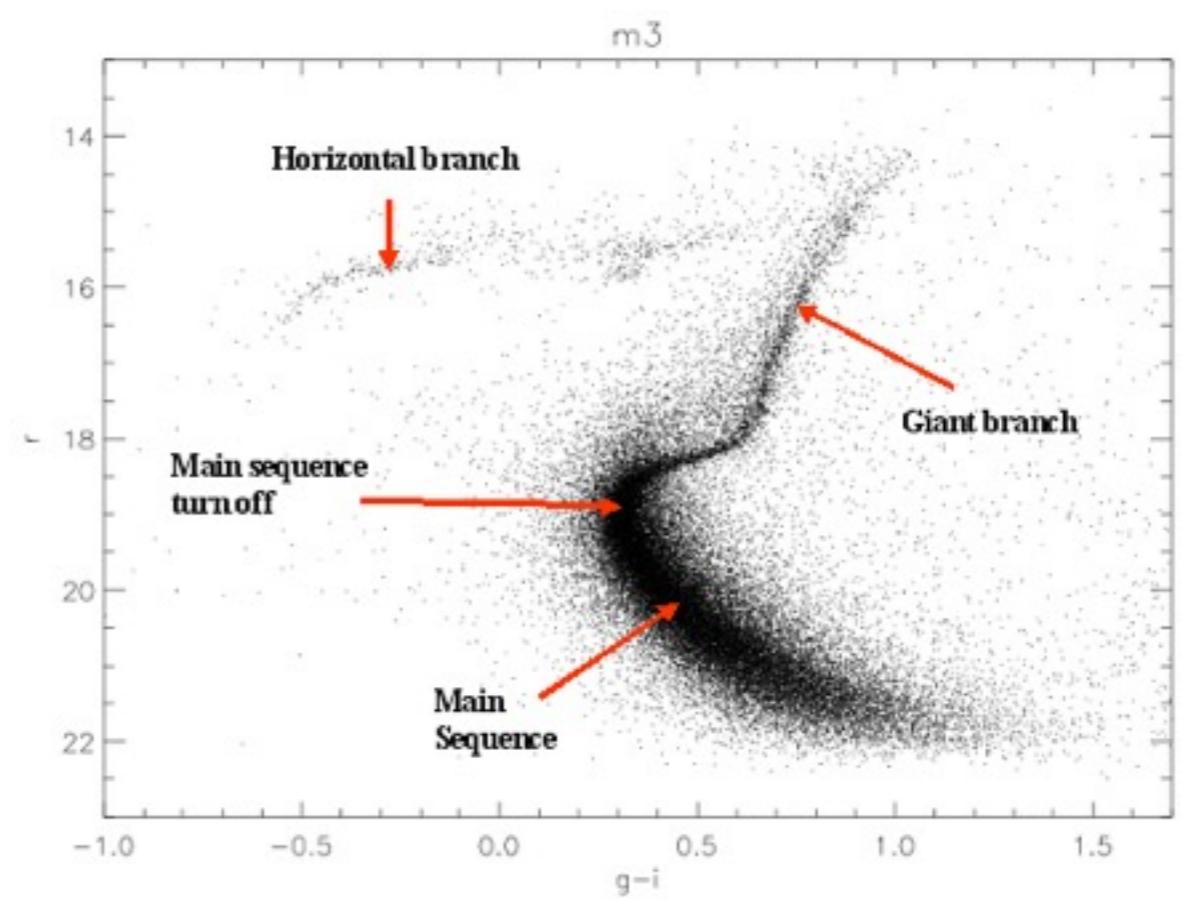
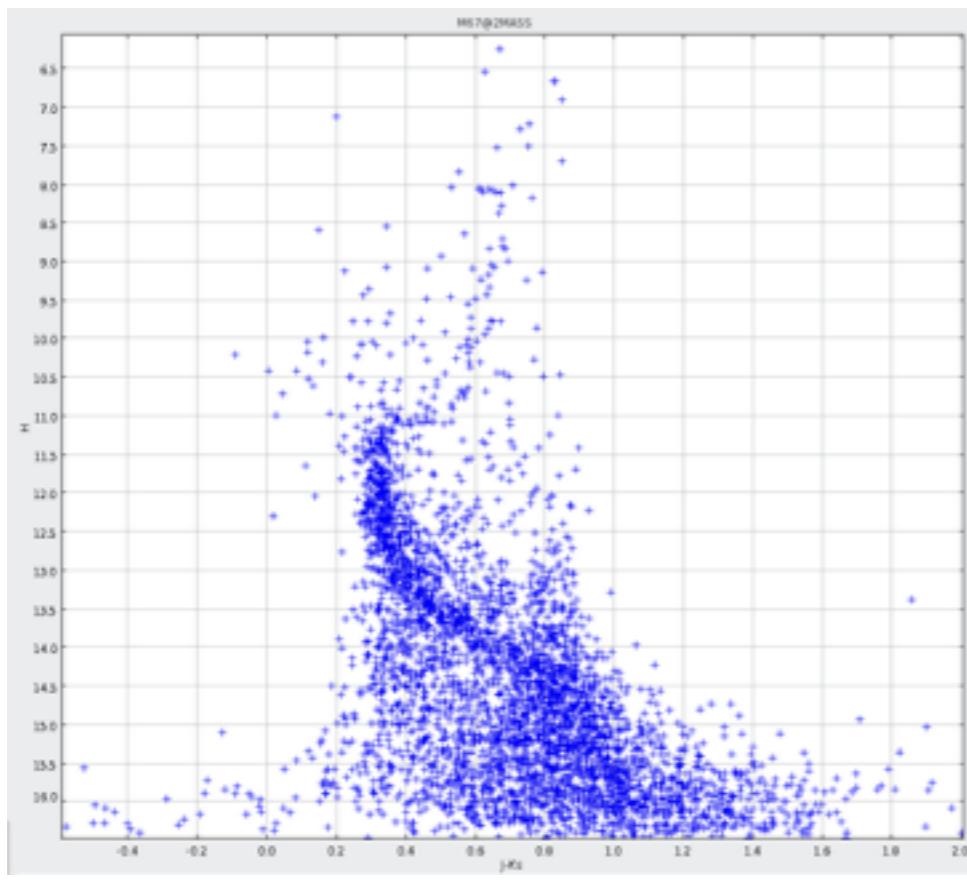
ABSTRACT

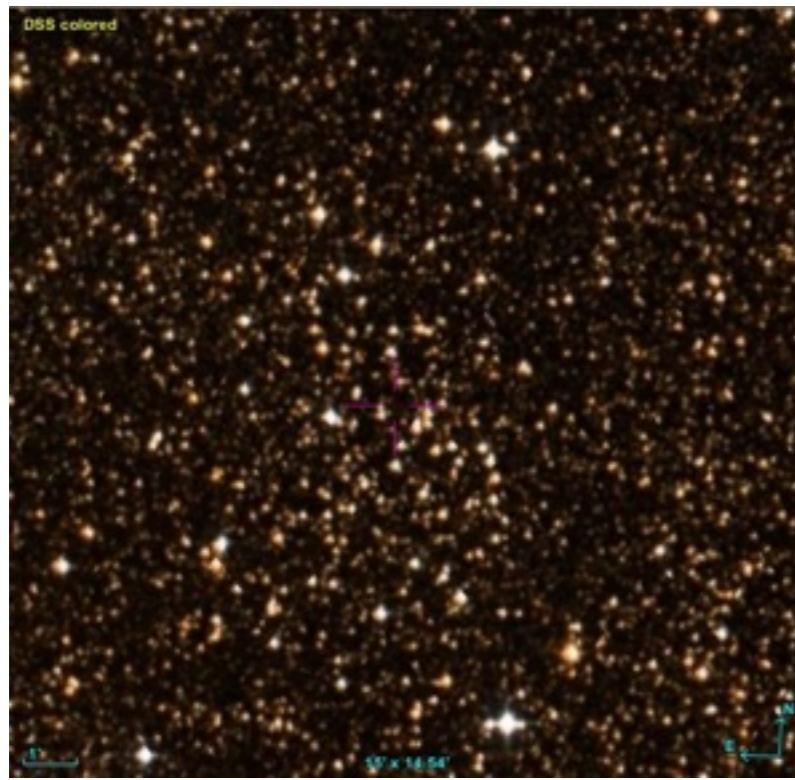
We present a new technique to fit color–magnitude diagrams of open clusters based on the cross-entropy global optimization algorithm. The method uses theoretical isochrones available in the literature and maximizes a weighted likelihood function based on distances measured in the color–magnitude space. The weights are obtained through a non parametric technique that takes into account the star distance to the observed center of the cluster, observed magnitude uncertainties, the stellar density profile of the cluster among others. The parameters determined simultaneously are distance, reddening, age and metallicity. The method takes binary fraction into account and uses a Monte-Carlo approach to obtain uncertainties on the determined parameters for the cluster by running the fitting algorithm many times with a re-sampled data set through a bootstrapping procedure. We present results for 9 well studied open clusters, based on 15 distinct data sets, and show that the results are consistent with previous studies. The method is shown to be reliable and free of the subjectivity of most previous visual isochrone fitting techniques.

Key words. open clusters and associations: general – methods: statistical

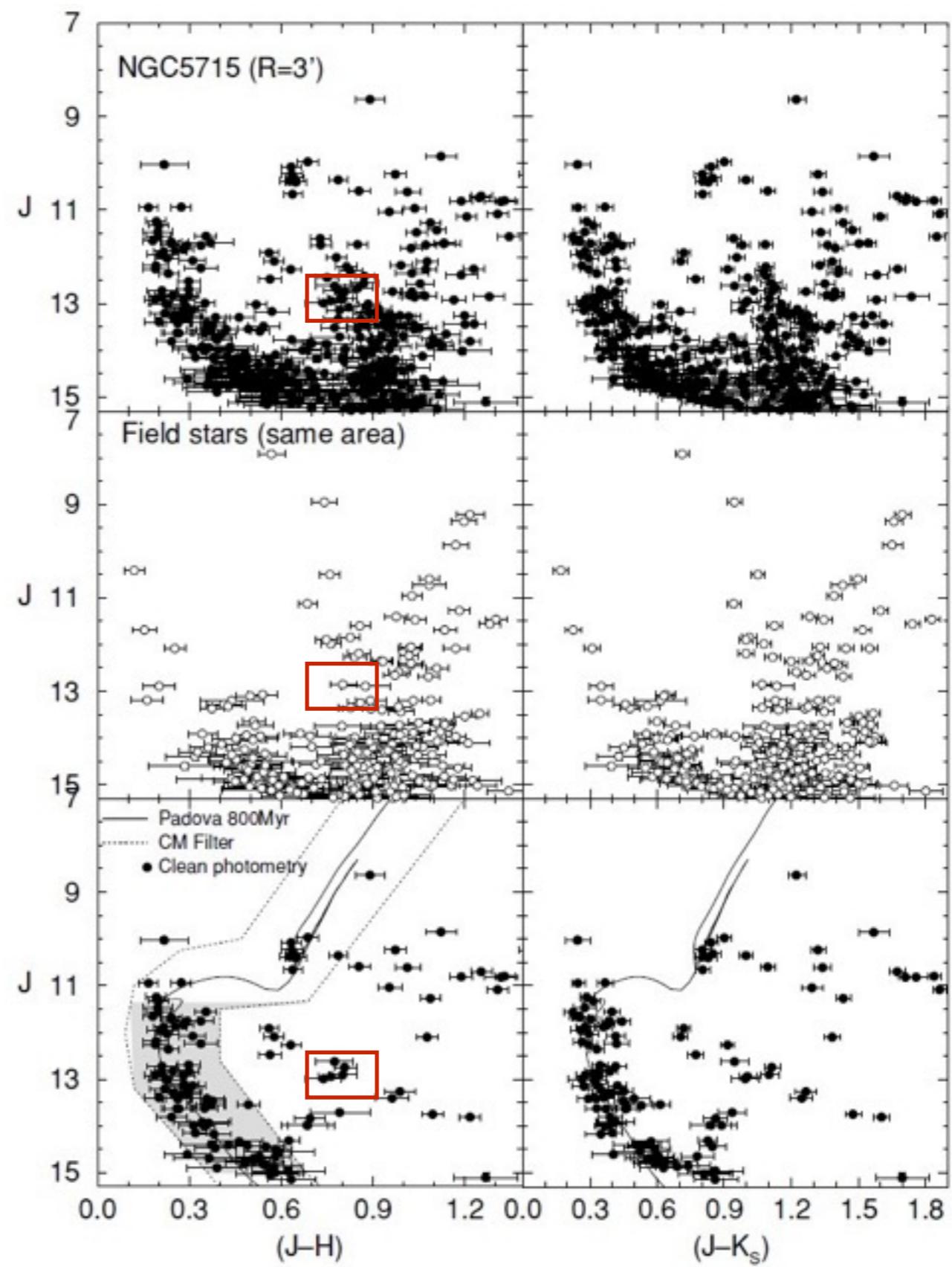
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Background





ref.Bonatto et al.
MNRAS,377,1301(2007)



Determining the likelihood

The weighted likelihood function is then given in the usual manner by:

$$\mathcal{L} = \prod_l^{N_d} P(V, BV, UB|I_N)_l \times W_l \quad (9)$$

$$\begin{aligned} P(V, BV, UB|I_N)_l &= \sum_m \frac{1}{\sigma_{V_l} \sigma_{BV_l} \sigma_{UB_l}} \\ &\times \text{EXP} \left[-\frac{1}{2} \left(\frac{V_l - I_{N,V_m}}{\sigma_{V_l}} \right)^2 \right] \\ &\times \text{EXP} \left[-\frac{1}{2} \left(\frac{BV_l - I_{N,BV_m}}{\sigma_{BV_l}} \right)^2 \right] \\ &\times \text{EXP} \left[-\frac{1}{2} \left(\frac{UB_l - I_{N,UB_m}}{\sigma_{UB_l}} \right)^2 \right] \end{aligned}$$

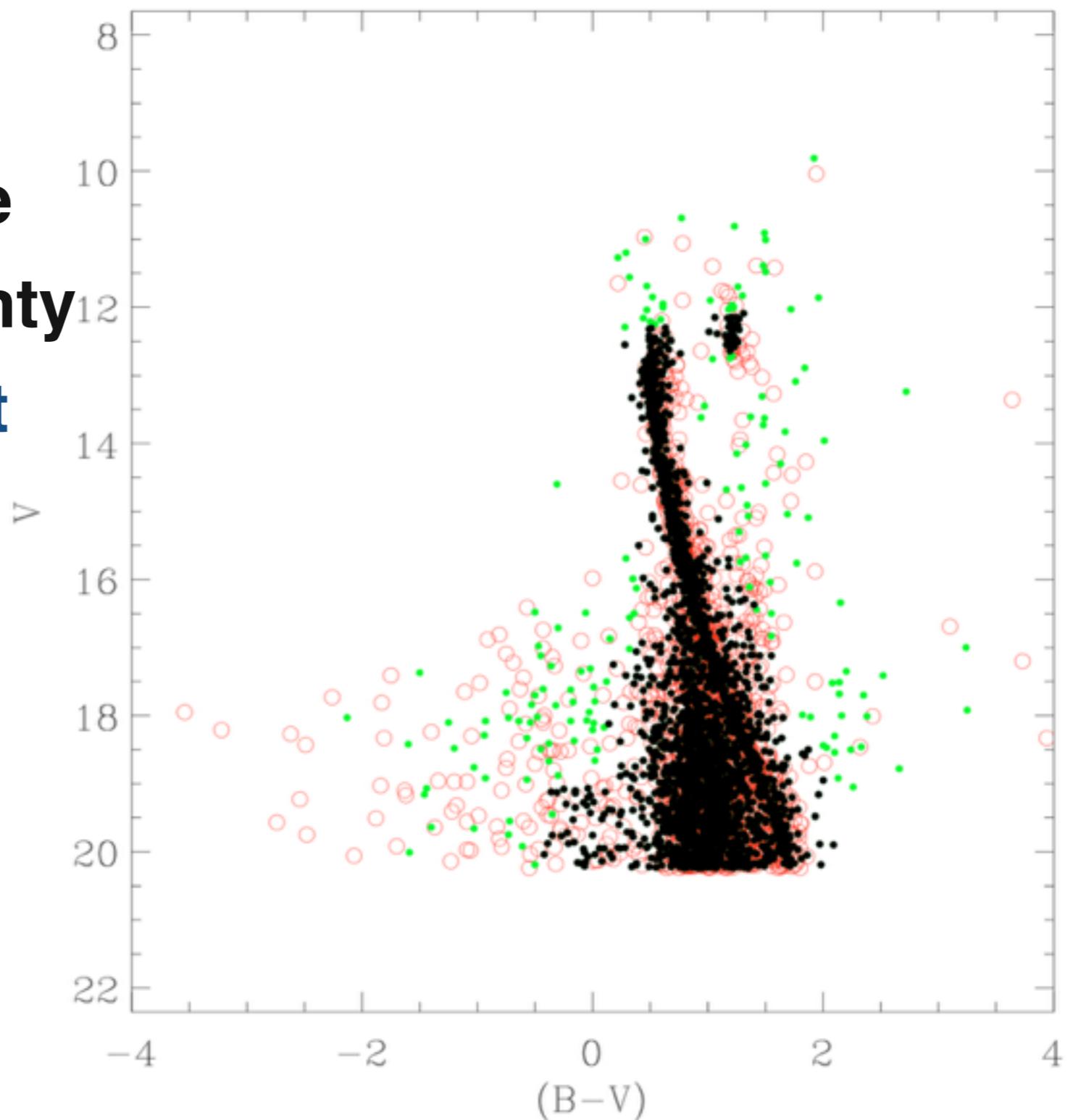
$$S(X) = -\log(\mathcal{L}(X))$$

cross entropy global optimization algorithm

Determining the weight function

- Magnitude cut-off
- Cluster density profile
- Photometric uncertainty
- Non parameter weight function

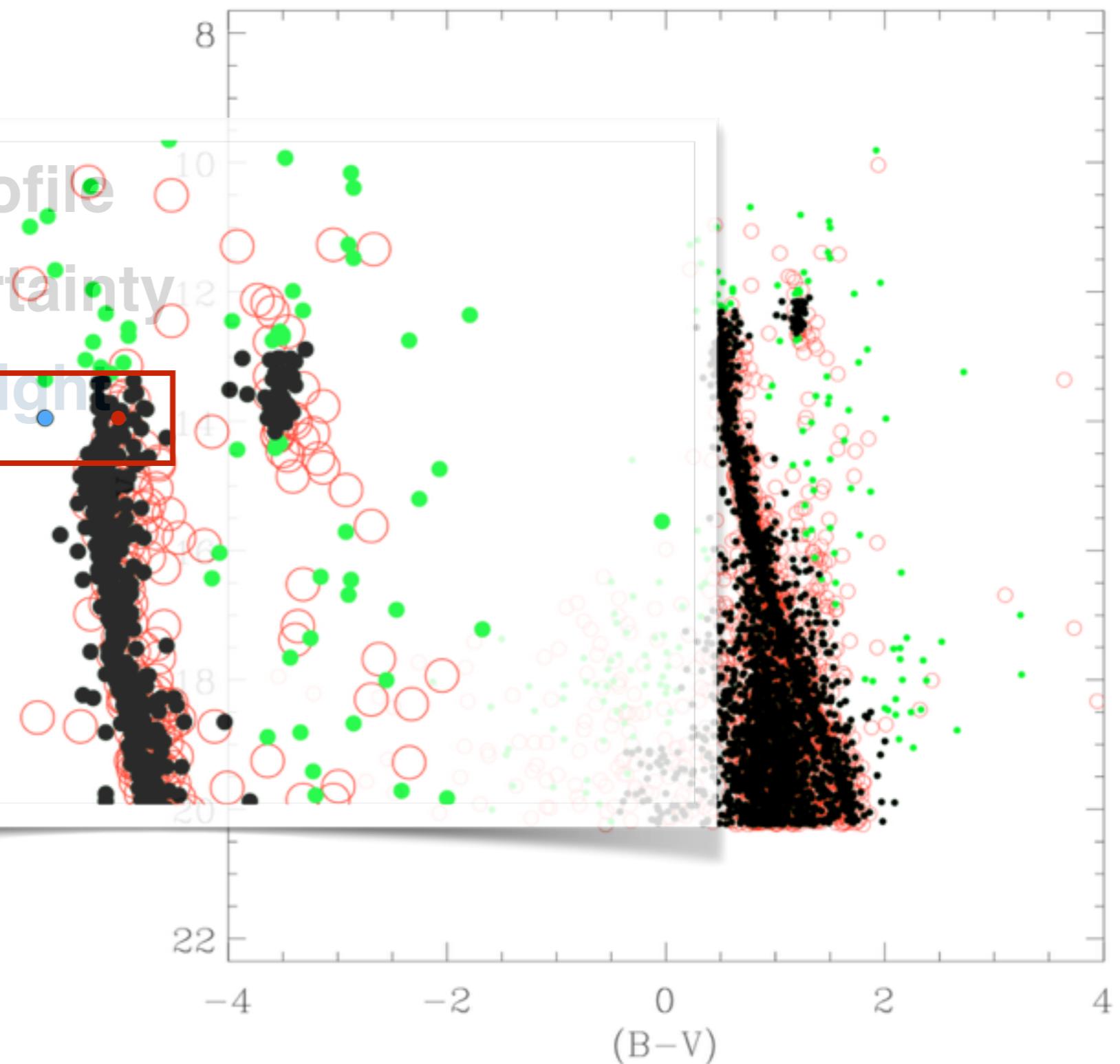
$$W_l = \frac{1}{\sigma_{V_l} \sigma_{BV_l} \sigma_{UB_l}} \times \text{EXP}^{\frac{-(V_l - \bar{V}_c)^2}{2\sigma_{V_c}^2}} \times \text{EXP}^{\frac{-(BV_l - \bar{BV}_c)^2}{2\sigma_{BV_c}^2}} \times \text{EXP}^{\frac{-(UB_l - \bar{UB}_c)^2}{2\sigma_{UB_c}^2}} \times \text{EXP}^{\frac{-r_l^2}{2\left(\frac{R_{\text{cluster}}}{3}\right)^2}}$$



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离重心越远， 权重越低

离星团中心越远，
权重越低

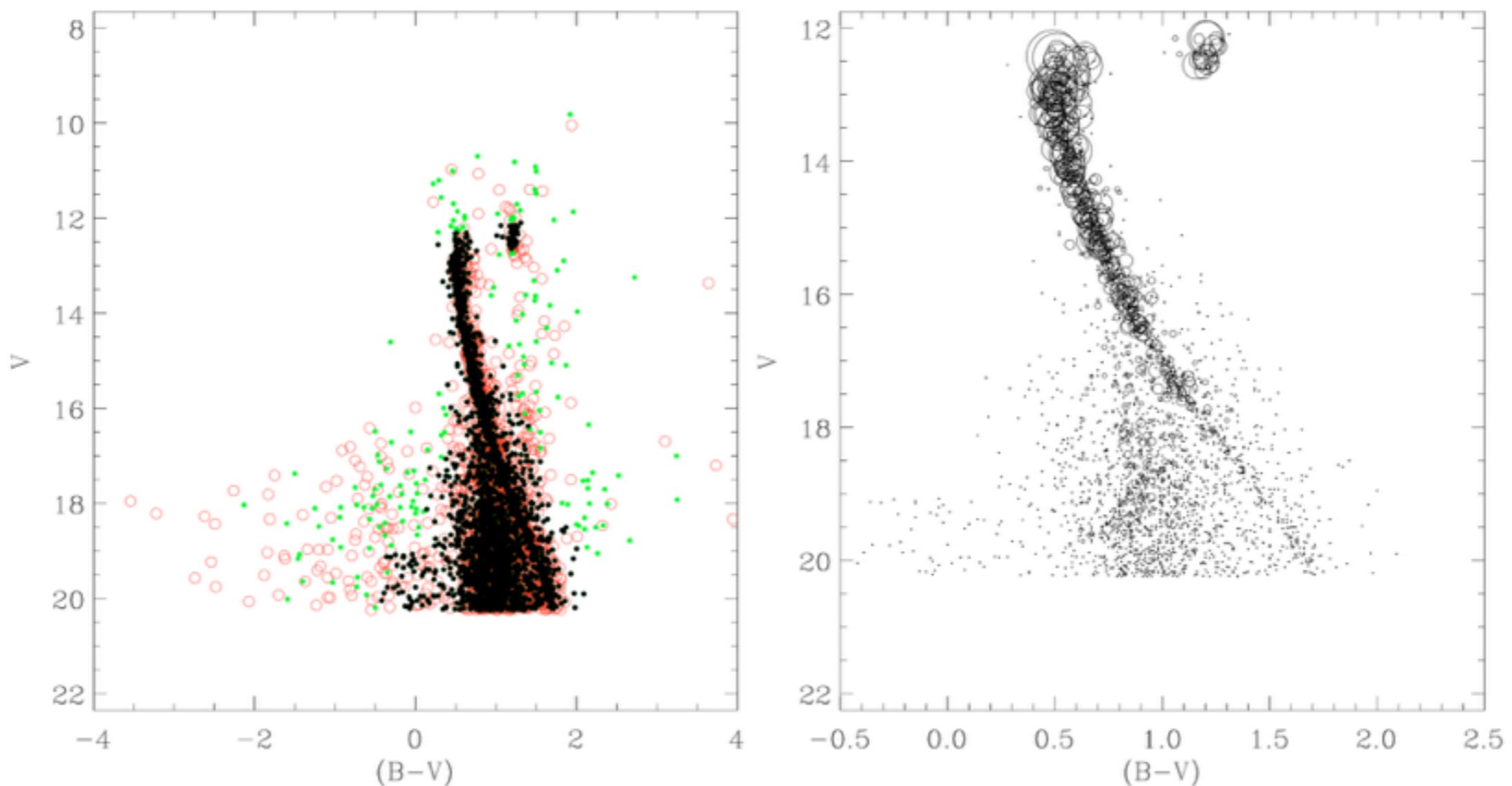


Fig. 1. Result of the decontamination (*left*) and weighting process (*right*) for the cluster NGC 2477. The black dots in the left graph are the selected stars after decontamination, the open circles are stars that fall outside the defined cluster radius and light dots are stars for which no statistic was available due to low numbers. The right graph shows open circles with sizes scaled to their weights, with larger sizes meaning larger weights.

Synthetic clusters

Table 1. Results for synthetic clusters studied by the fitting method.

Cluster	N_{stars}	Contamination (%)	$3\sigma_{\text{phot}}(\%)$	$E(B - V)(\text{mag})$	Distance (pc)	$\log(Age) (\text{yr})$
SC 01	432	0%	1.0	0.40 ± 0.01	2112 ± 51	8.65 ± 0.05
SC 02	480	20%	1.0	0.40 ± 0.01	2062 ± 43	8.71 ± 0.05
SC 03	444	50%	1.0	0.38 ± 0.01	2073 ± 30	8.70 ± 0.07
SC 04	65	0%	1.0	0.38 ± 0.02	2008 ± 94	8.70 ± 0.07
SC 05	113	20%	1.0	0.40 ± 0.03	2060 ± 60	8.75 ± 0.06
SC 06	61	50%	1.0	0.40 ± 0.02	2102 ± 58	8.70 ± 0.07

Notes. Synthetic clusters were generated with parameters $\log(\text{age}) = 8.70 \text{ yr}$, distance = 2100 pc, $E(B - V) = 0.40$, $Z = 0.019$ and N_{stars} , including the given contamination fraction and photometric accuracy of $3\sigma_{\text{phot}}$.

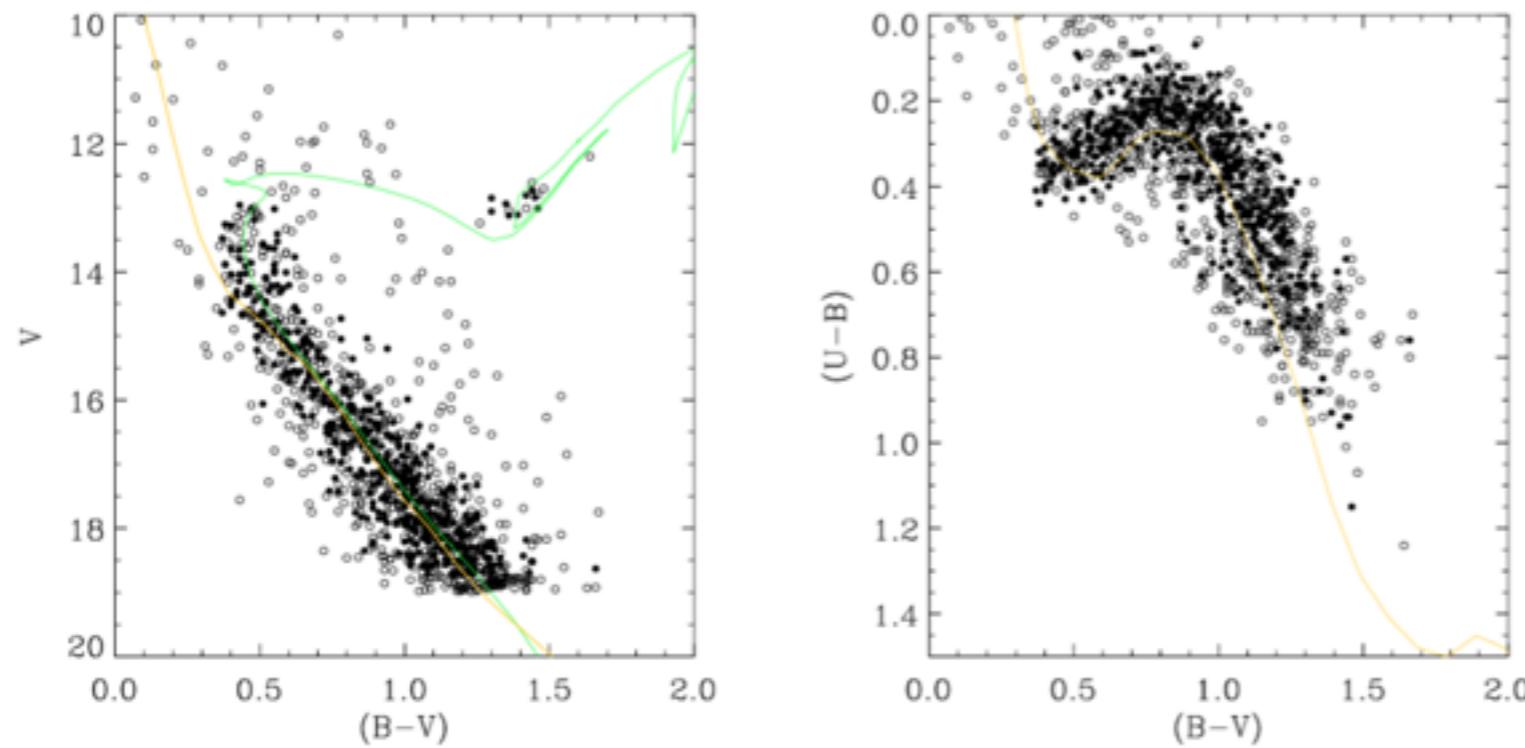


Fig. 4. Result of the fitting method for the synthetic cluster SC 03 with the ZAMS (thin line) and the fitted isochrone (thick line) where we show the rejected stars by our filtering method (open circles) as well as stars used in the fit (filled circles).

Fitting published data

Table 3. Basic parameters obtained for the investigated clusters.

Cluster	Fit					Literature			Ref.
	$E(B - V)$ (mag)	Distance (pc)	Log(Age) (yr)	Z	$E(B - V)$ (mag)	distance (pc)	Log(Age) (yr)	Ref.	
NGC 2477	0.29 ± 0.03	1385 ± 64	8.90 ± 0.09	0.019	0.26 ± 0.08	1227 ± 166	8.94 ± 0.11	152	
NGC 7044	0.55 ± 0.05	3093 ± 345	9.35 ± 0.17	0.019	0.63 ± 0.06	3097 ± 145	9.26 ± 0.08	62	
NGC 2266	0.17 ± 0.02	3100 ± 244	8.90 ± 0.07	0.008	0.10 ± 0.01	3490 ± 180	8.87 ± 0.04	41	
Berkeley 32	0.12 ± 0.04	3483 ± 186	9.65 ± 0.13	0.008	0.15 ± 0.03	3491 ± 401	9.54 ± 0.08	40	
NGC 2682	0.02 ± 0.01	774 ± 25	9.55 ± 0.05	0.019	0.05 ± 0.02	820 ± 47	9.61 ± 0.09	335	
	0.05 ± 0.01	758 ± 26	9.60 ± 0.06	0.019	0.05 ± 0.02	820 ± 47	9.61 ± 0.09	31	
	0.04 ± 0.01	869 ± 57	9.50 ± 0.07	0.019	0.05 ± 0.02	820 ± 47	9.61 ± 0.09	54	
NGC 2506	0.03 ± 0.01	3587 ± 198	9.20 ± 0.05	0.008	0.06 ± 0.04	3315 ± 219	9.22 ± 0.11	284	
	0.07 ± 0.01	3137 ± 177	9.25 ± 0.05	0.008	0.06 ± 0.04	3315 ± 219	9.22 ± 0.11	163	
NGC 2355	0.25 ± 0.02	2316 ± 103	8.80 ± 0.05	0.008	0.14 ± 0.06	2086 ± 163	8.92 ± 0.07	217	
	0.19 ± 0.02	2022 ± 88	8.85 ± 0.05	0.008	0.14 ± 0.06	2086 ± 163	8.92 ± 0.07	44	
Melotte 105	0.47 ± 0.02	1750 ± 111	8.40 ± 0.06	0.019	0.48 ± 0.05	2094 ± 159	8.35 ± 0.09	289	
	0.49 ± 0.03	2005 ± 139	8.45 ± 0.07	0.019	0.48 ± 0.05	2094 ± 159	8.35 ± 0.09	32	
Trumpler 1	0.59 ± 0.05	2419 ± 185	7.85 ± 0.19	0.019	0.57 ± 0.04	2356 ± 511	7.48 ± 0.08	320	
	0.53 ± 0.03	2309 ± 121	7.55 ± 0.31	0.019	0.57 ± 0.04	2356 ± 511	7.48 ± 0.08	86	

Notes. The numbers, in the last column are the WEBDA reference codes, $E(B - V)$ is the extinction, d the distance to the cluster, $\log(Age)$ the logarithm of the age (in years), Z the adopted metallicity. The literature values are those of [Paunzen & Netopil \(2006\)](#).

But...

Determining the likelihood

The weighted likelihood function is then given in the usual manner by:

$$\mathcal{L} = \prod_l^{N_d} P(V, BV, UB|I_N)_l \times W_l \quad \longleftarrow \text{权重在这里是个常数, 和参数无关}$$

$$\begin{aligned} P(V, BV, UB|I_N)_l &= \sum_m \frac{1}{\sigma_{V_l} \sigma_{BV_l} \sigma_{UB_l}} \\ &\times \exp \left[-\frac{1}{2} \left(\frac{V_l - I_{N,V_m}}{\sigma_{V_l}} \right)^2 \right] \\ &\times \exp \left[-\frac{1}{2} \left(\frac{BV_l - I_{N,BV_m}}{\sigma_{BV_l}} \right)^2 \right] \\ &\times \exp \left[-\frac{1}{2} \left(\frac{UB_l - I_{N,UB_m}}{\sigma_{UB_l}} \right)^2 \right] \end{aligned} \quad \longleftarrow \text{误差不是独立的, 未考虑协方差项}$$

$$S(X) = -\log(\mathcal{L}(X))$$

Determining the weight function

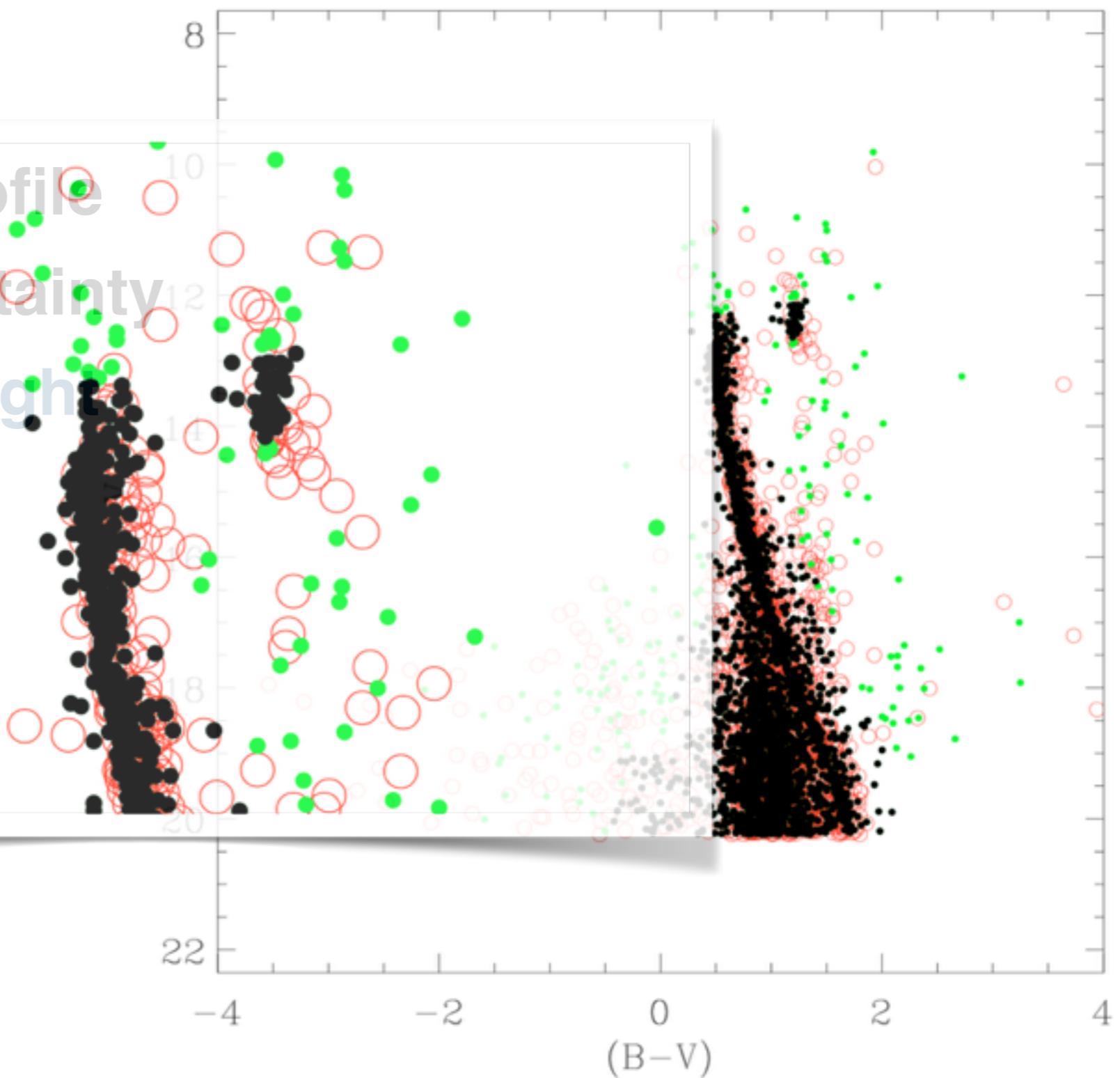
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并不高斯

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$$\frac{\sum_i V_{C_i} - \sum_i V_{F_i}}{N_C - N_F} = \bar{V}$$

