Primordial Gravitational Waves & the CMB Physics

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Hefei, March 11th 2018

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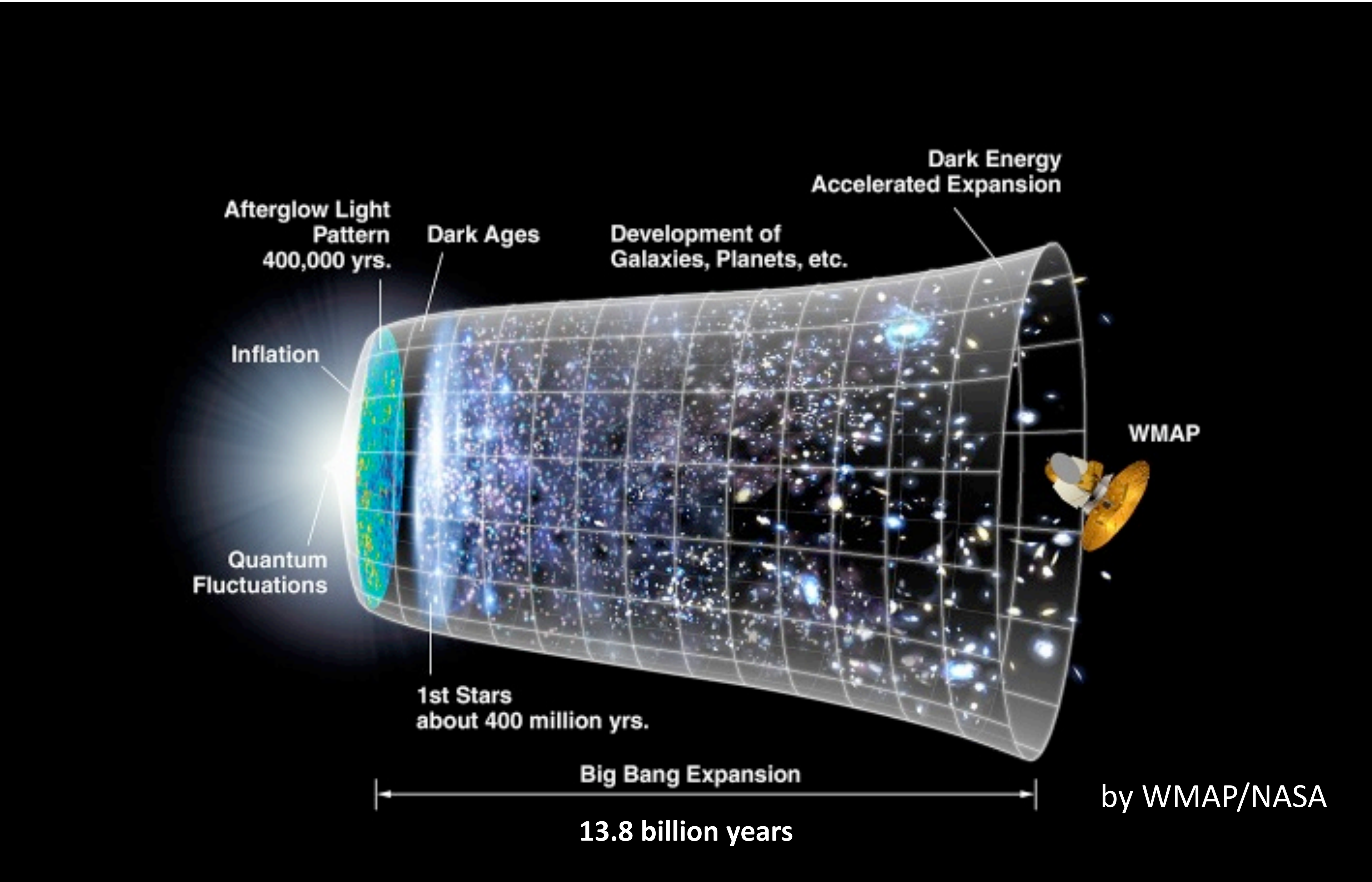
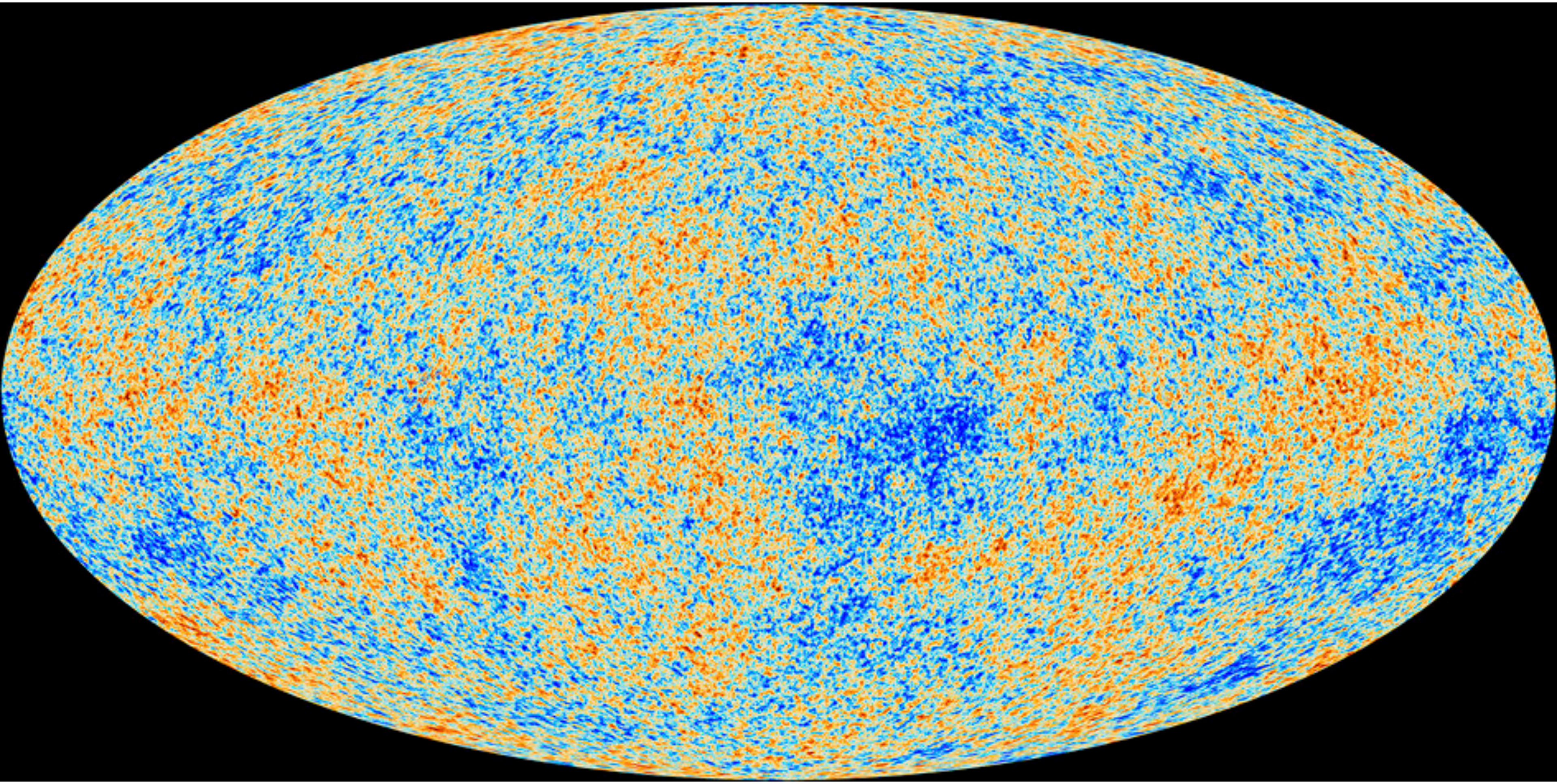
hot Big Bang & Inflation(Guth, Starobinsky, Sato, Fang, 1980s)

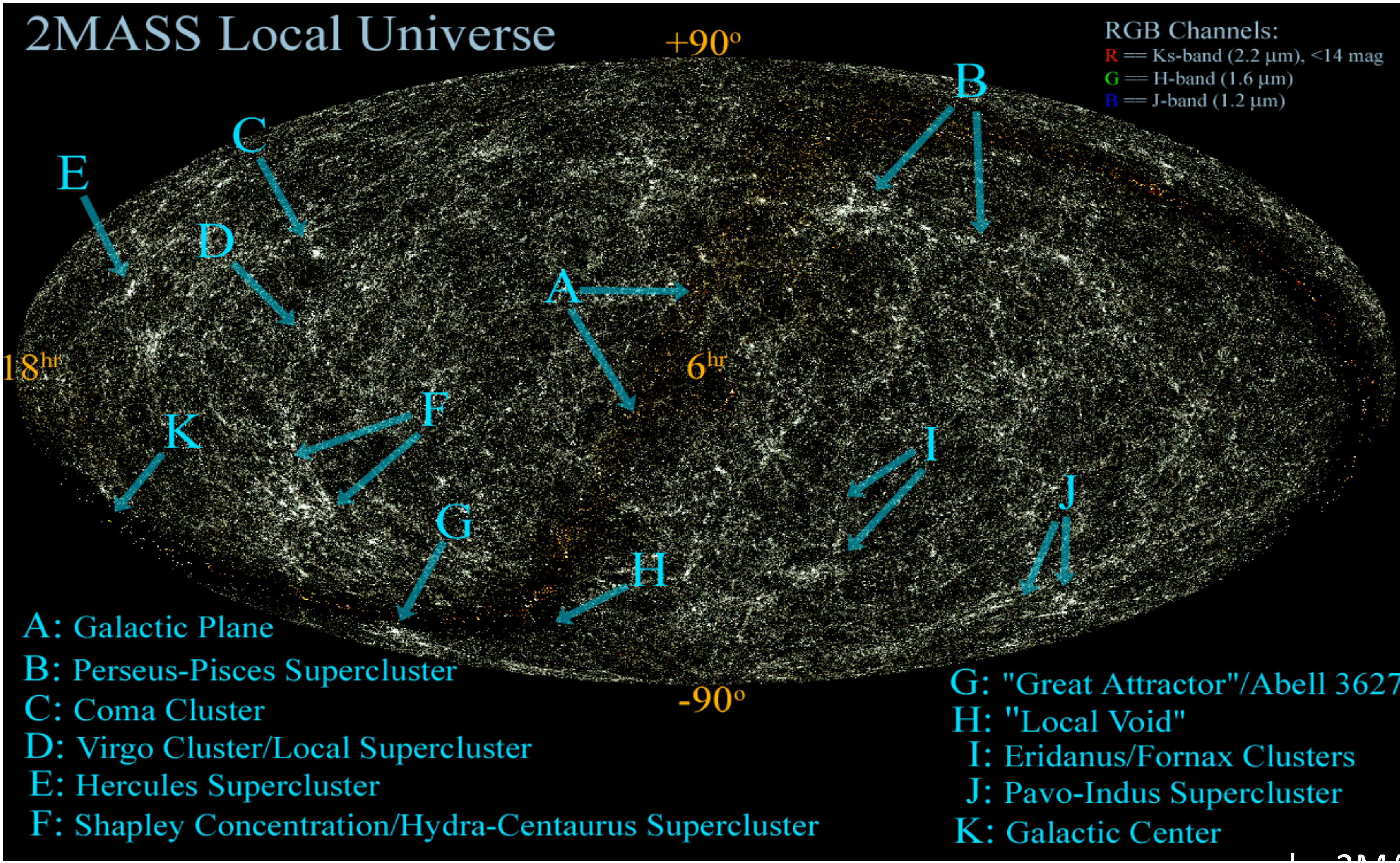


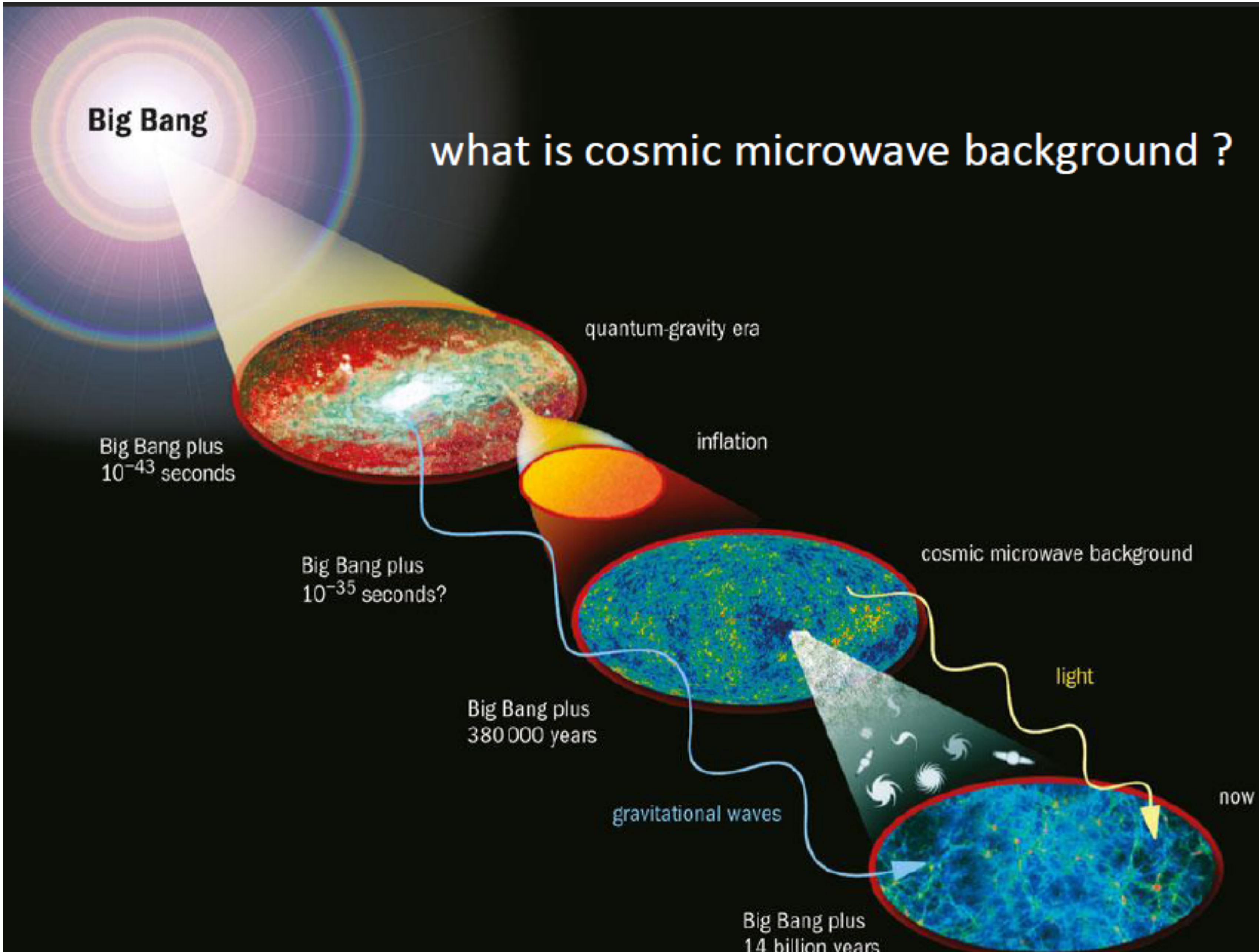
by WMAP/NASA

Cosmic Microwave Background (CMB) Anisotropies

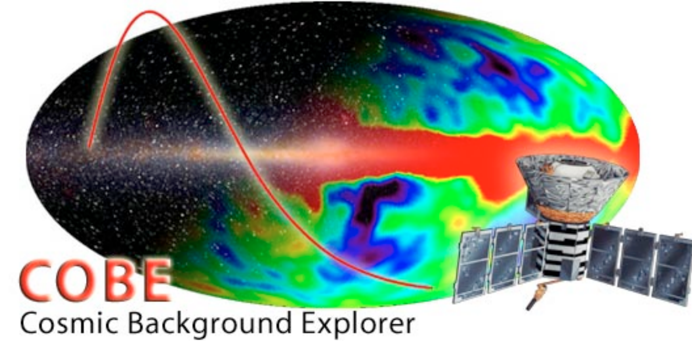


Large Scale Structure (LSS) Survey




Big Bang
what is cosmic microwave background ?
quantum-gravity era
Big Bang plus
10^{-43} seconds
inflation
Big Bang plus
10^{-35} seconds?
cosmic microwave background
light
Big Bang plus
380 000 years
now
gravitational waves
Big Bang plus
14 billion years

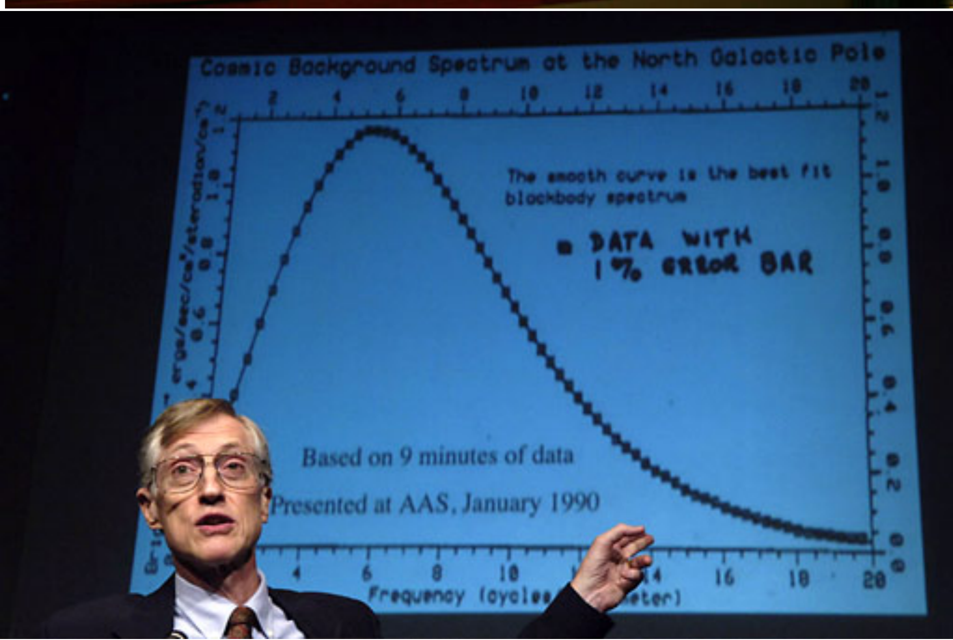
The Nobel Prize in Physics 2006

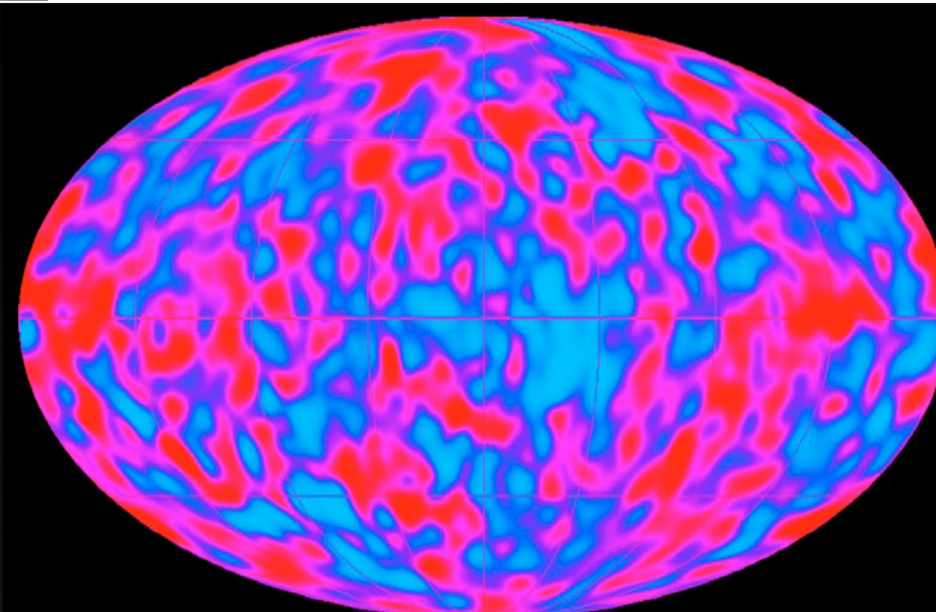


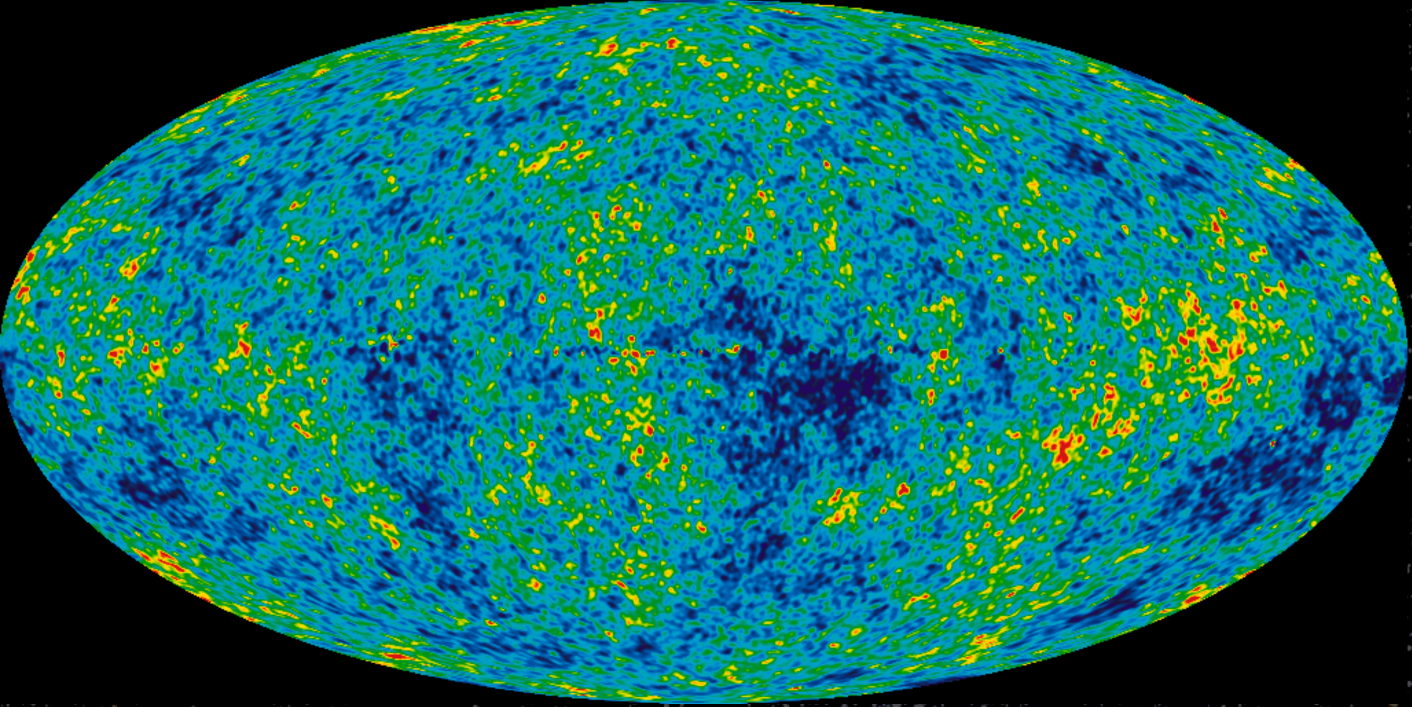
John C. Mather

George F. Smoot

"for their discovery of the ***blackbody form*** and ***anisotropy*** of the cosmic microwave background radiation"

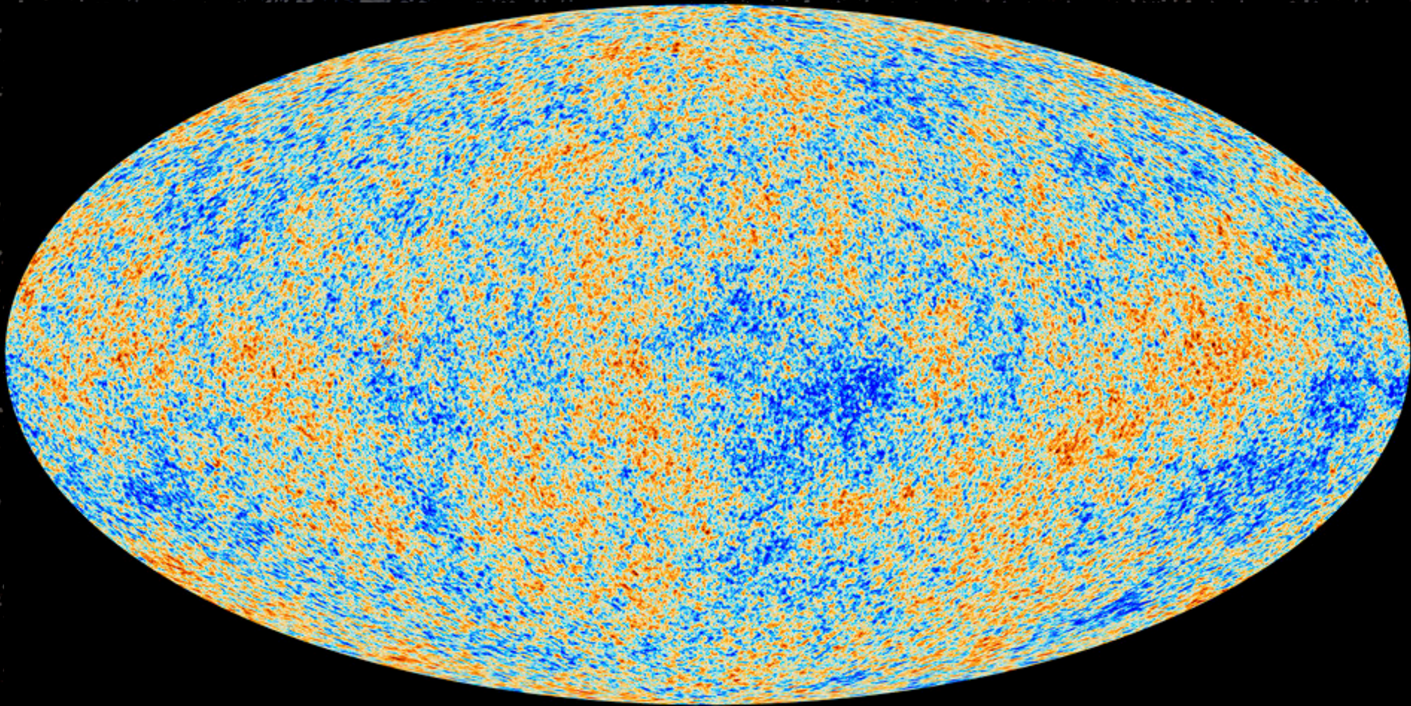


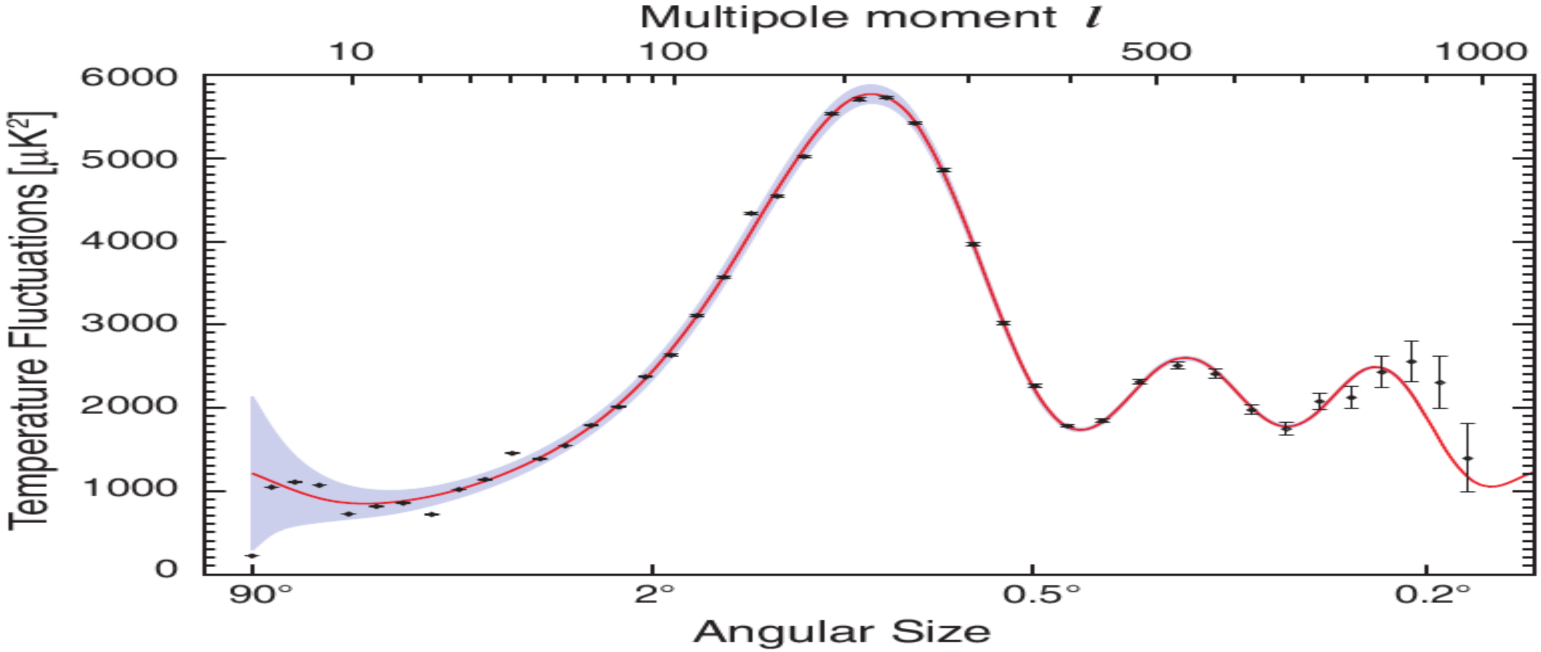




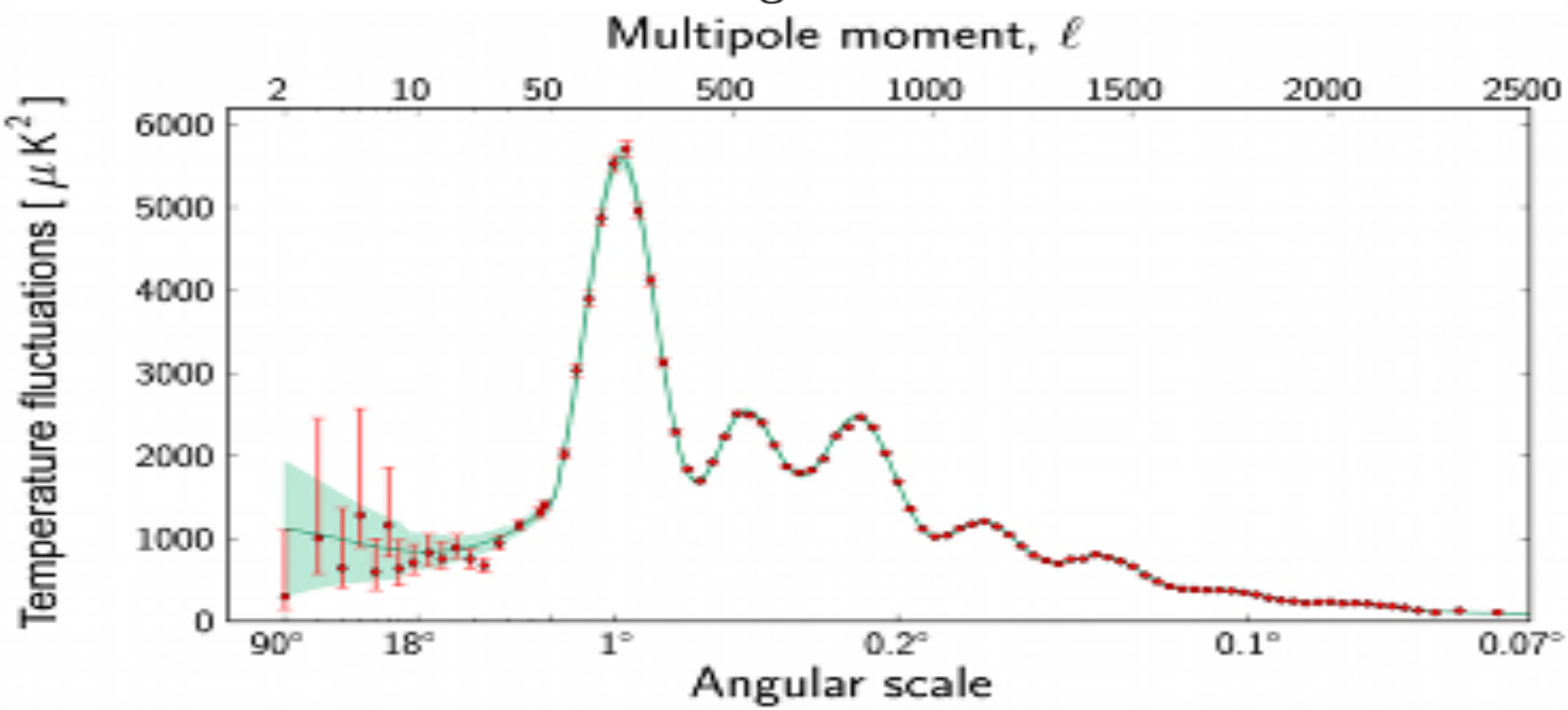
WMAP/NASA

Planck/ESA





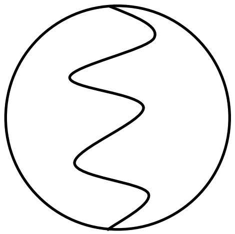
Multipole moment l
10
100
500
1000
6000
5000
4000
3000
2000
1000
0
Temperature Fluctuations [μK²]
90°
2°
0.5°
0.2°
Angular Size



Multipole moment, ℓ
2
10
50
500
1000
1500
2000
2500
6000
5000
4000
3000
2000
1000
0
Temperature fluctuations [μK²]
90°
18°
1°
0.2°
0.1°
0.07°
Angular scale

Primordial origin of temperature fluctuations

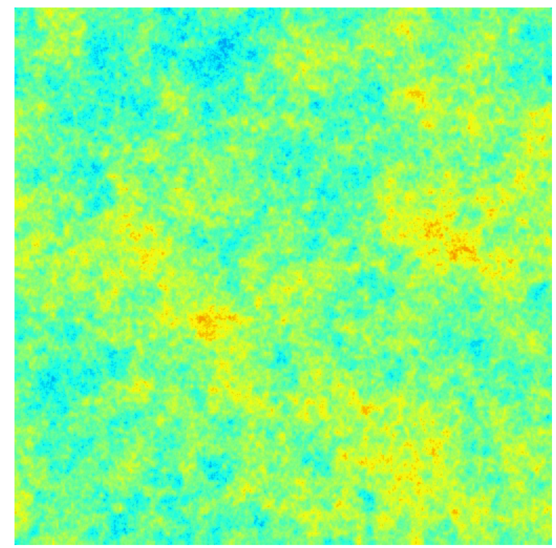
A first glance at perturbation theory in inflation



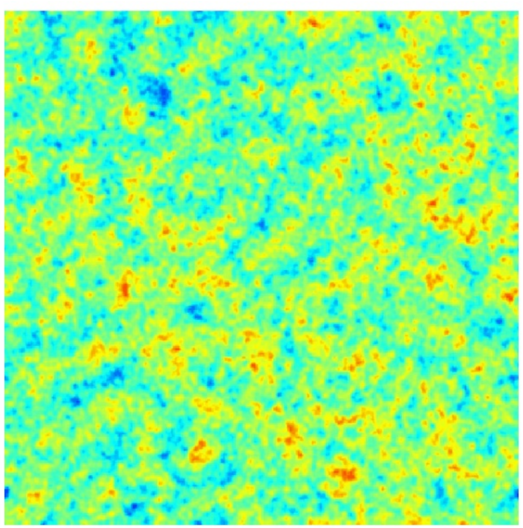
Quantum fluctuations
“waves in a pocket universe”

Inflation
$>10^{30}$ times bigger

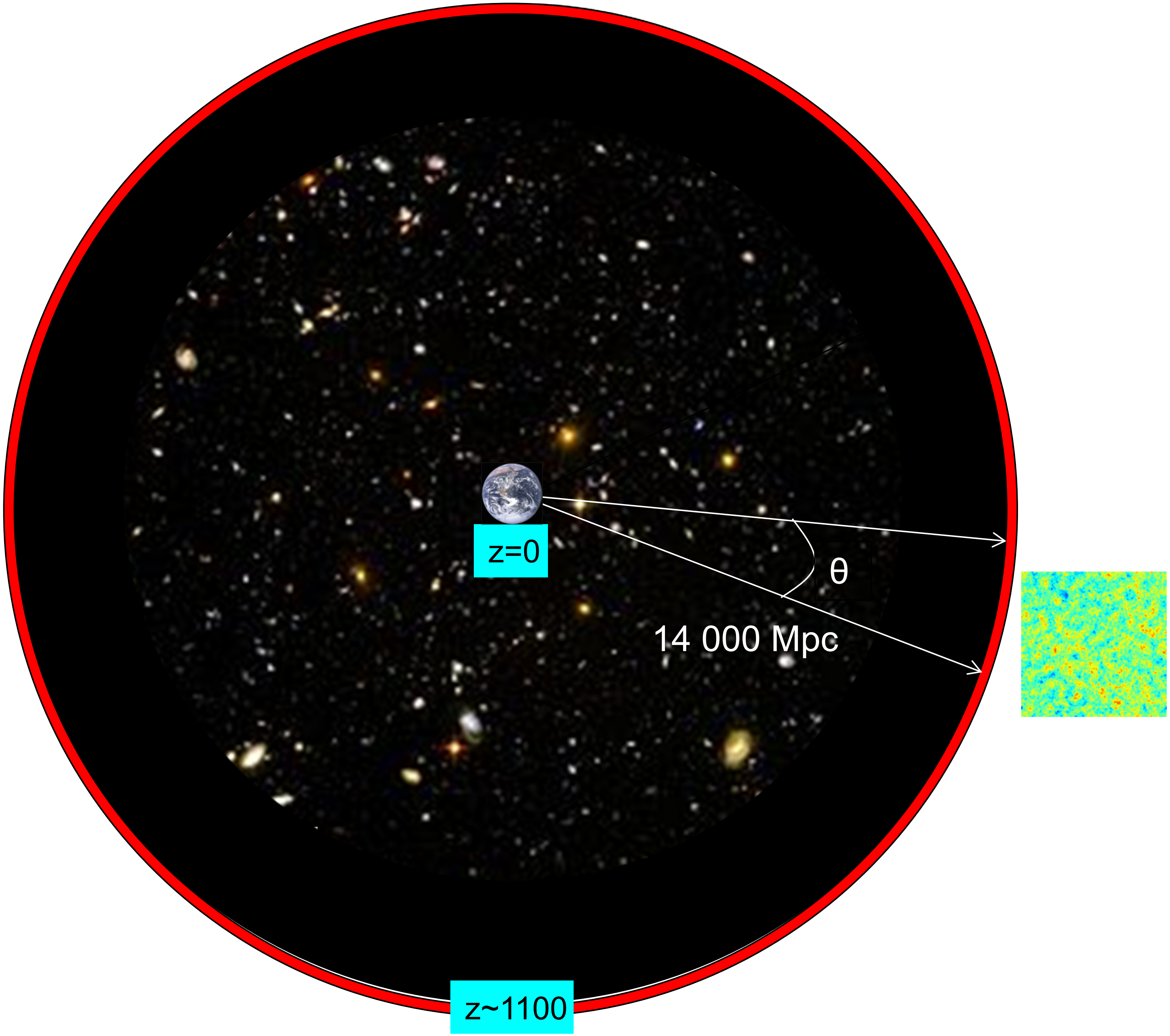
End of inflation
Huge size, amplitude $\sim 10^{-5}$



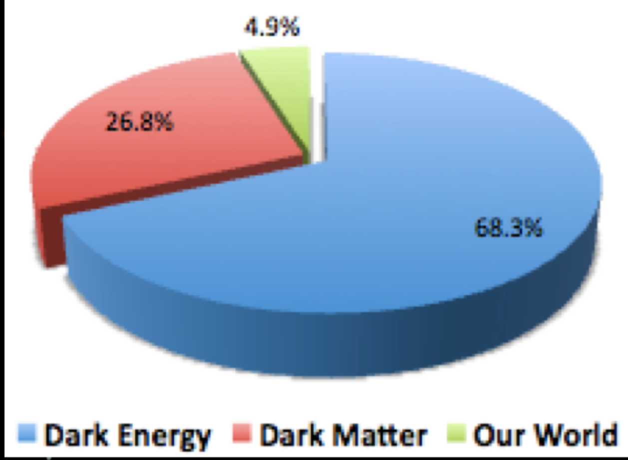
+ gravity
+ pressure
+ diffusion



Last scattering surface



z=0
θ
14 000 Mpc
z~1100



Observational Facts:

- Our universe has a thermal expanding history with 13.8 billion years

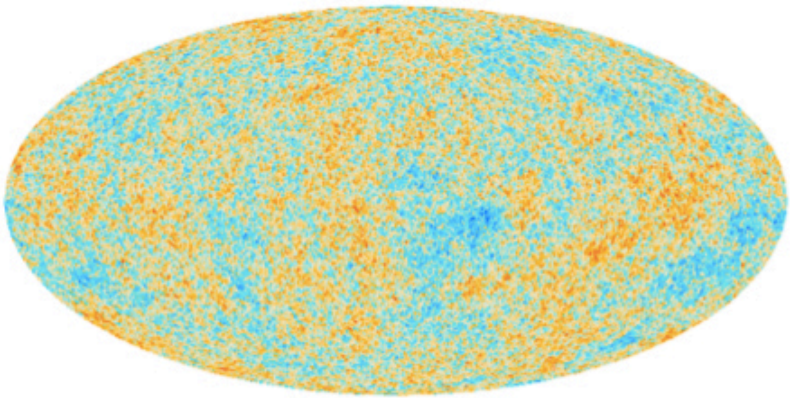
- The background looks the same at anywhere on sufficiently large scales

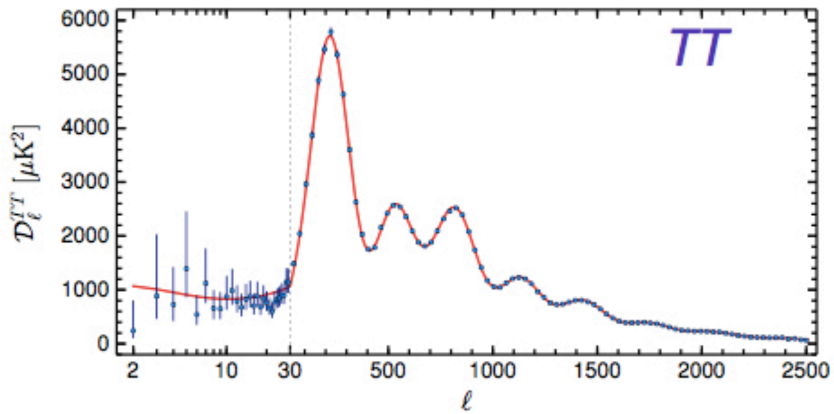
- Galaxies and clusters are basic blocks to form the LSS

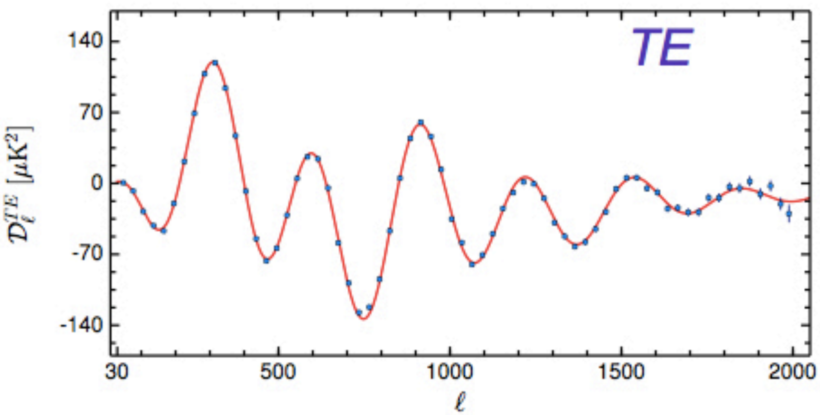
CMB is super important to the study of cosmology

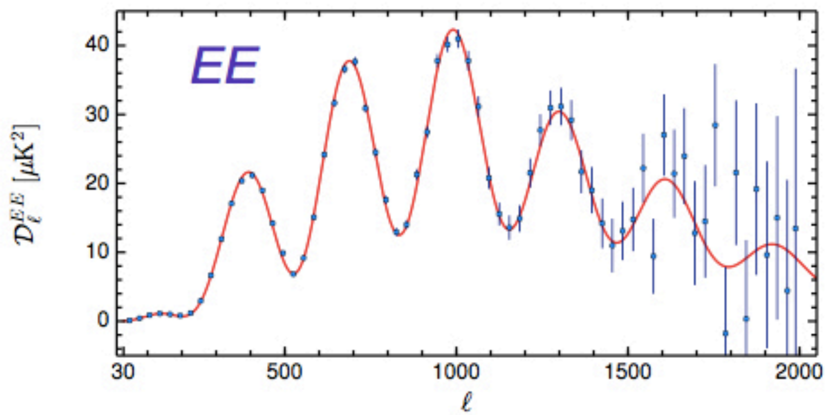
A novel lesson from Planck

CMB maps from Planck 2015





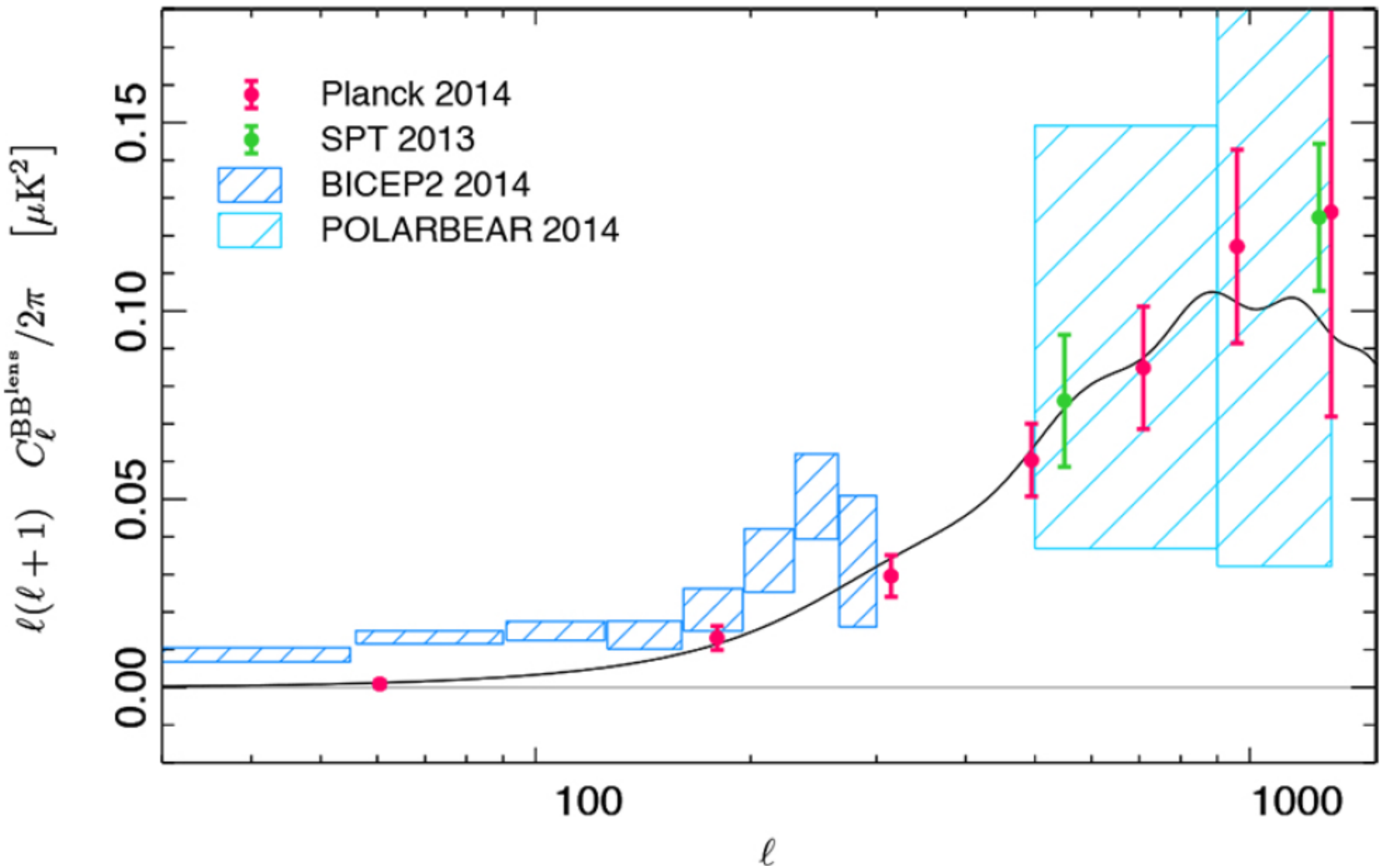




1502.01589

What can we learn from Planck 2015

Lensing induced B mode power spectrum:

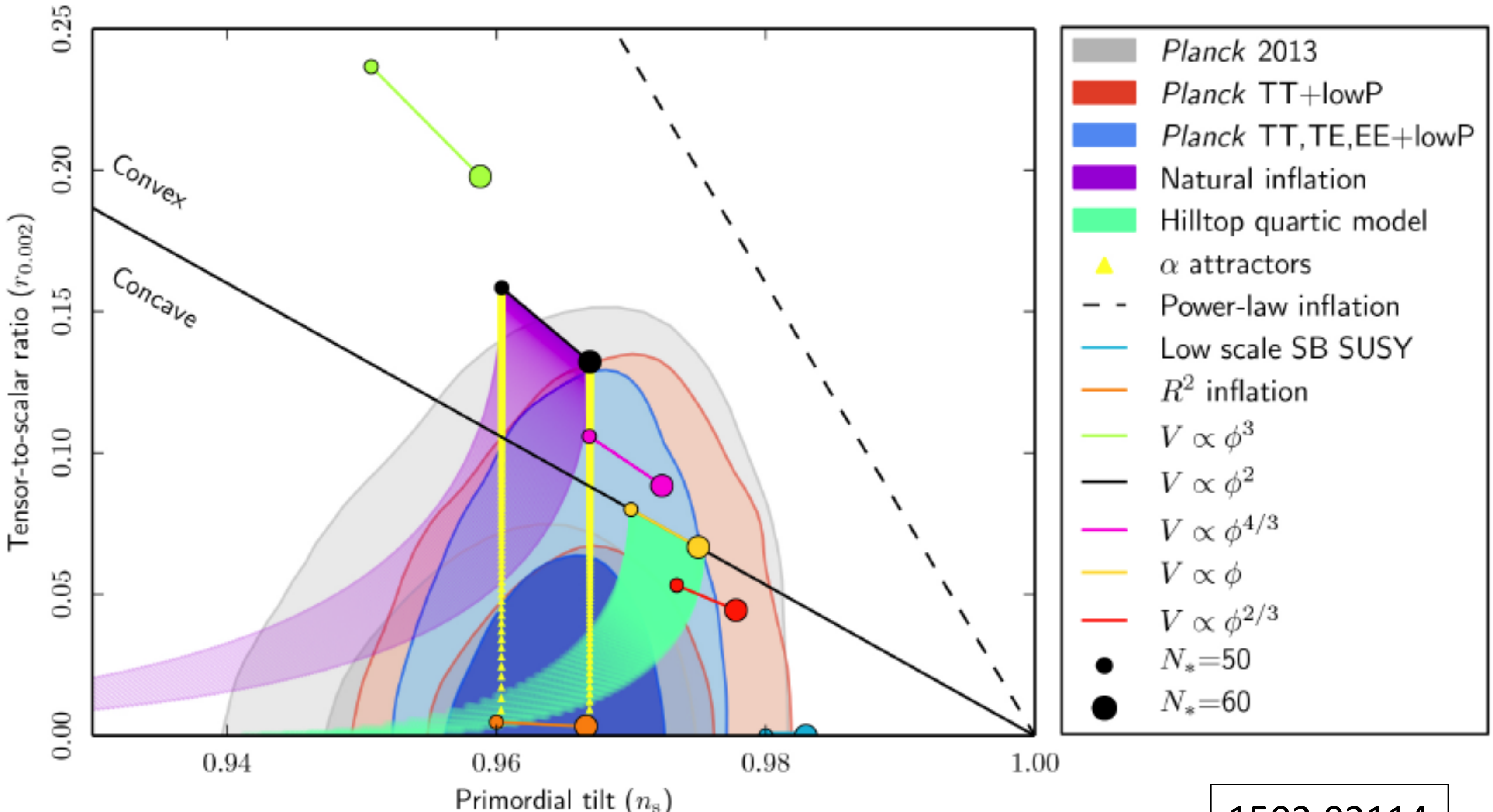


Concordance model: inflationary LCDM

$$\{H_0, \Omega_b, \Omega_c, A_s, n_s, \tau\}$$

- 7 peaks in 2013, 19 peaks in 2015;
- LCDM is perfect in explaining three CMB maps from l =30 until l=2000;
- A nearly scale-invariant, adiabatic, Gaussian power spectrum of primordial fluctuations as predicted by inflation seems highly consistent with data.

Concordance model: inflationary LCDM



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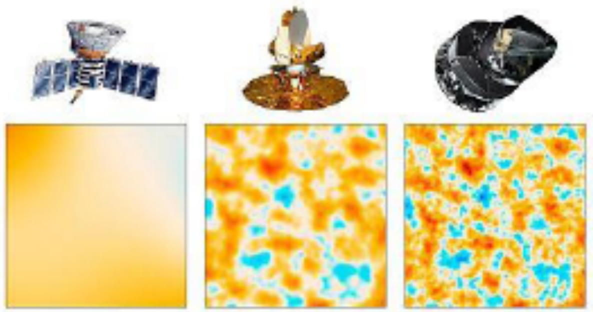
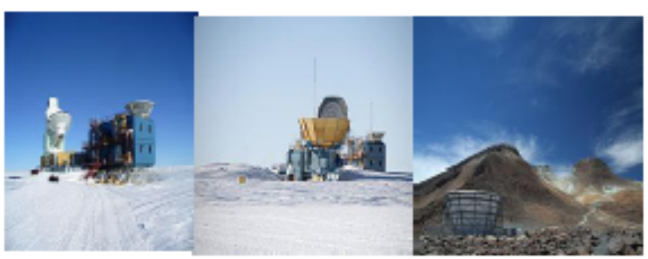
Planck 2015 data severely constrains the parameter space of inflationary cosmology.

CMB leads to the precision cosmology



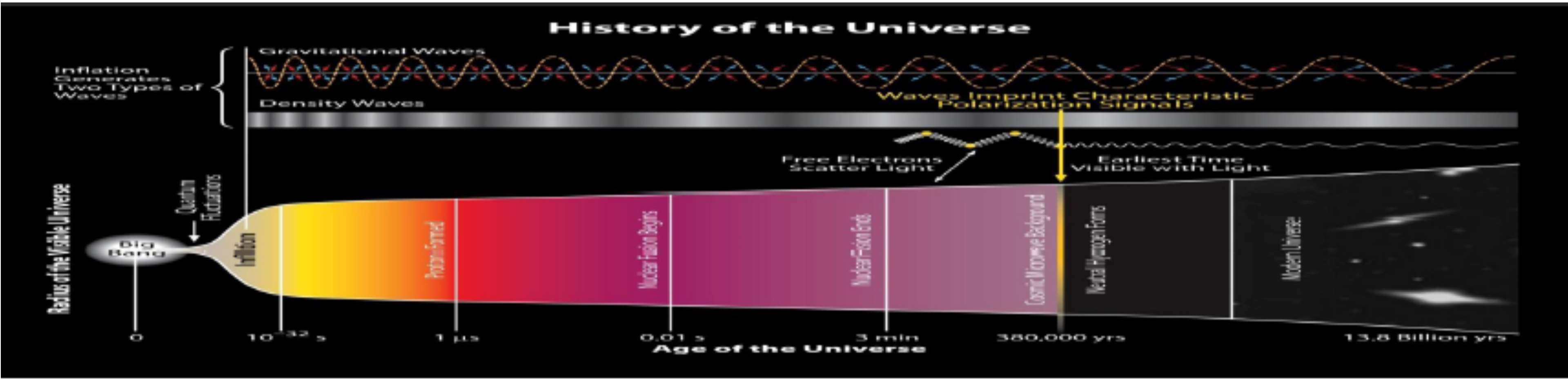
COBE WMAP Planck

- 1998, cosmic acceleration: top ten breakthrough of 《science》
- 2003, CMB Involved in top ten breakthrough
- 1978 and 2006, Nobel prize
- 2011, cosmic acceleration win Nobel
- 2010 and 2012, WMAP win Shao's prize and Gruber prize

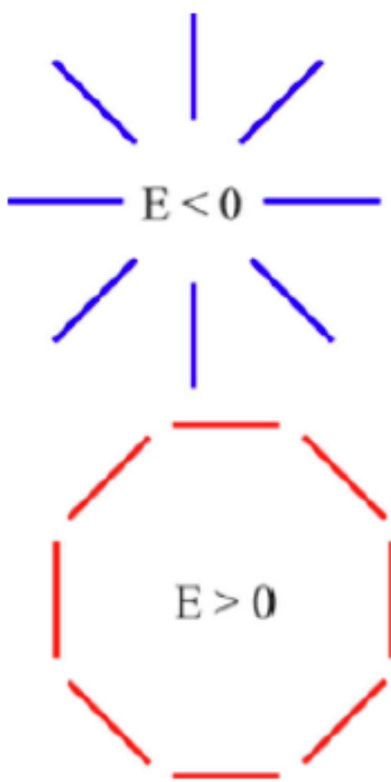


SPT BICEP/ keck Array ACT

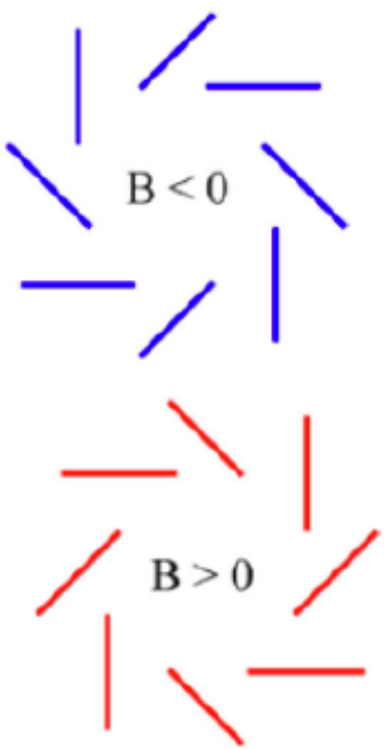
CMB polarization is significant in next generation measurements



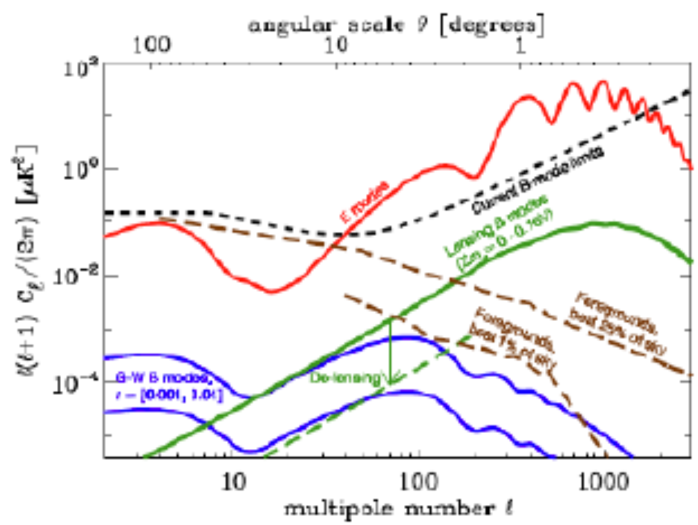
E mode



B mode



B mode survey can discriminate early universe models, namely, inflation; as well as testing the fundamental symmetries, such as CPT.



Shocking (but old) news !

The Nobel Prize in Physics 2017
Rainer Weiss, Barry C. Barish, Kip S. Thorne

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The Nobel Prize in Physics 2017

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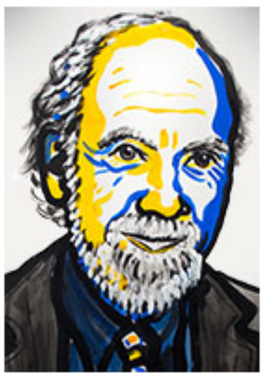
Rainer Weiss
Prize share: 1/2



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Barry C. Barish
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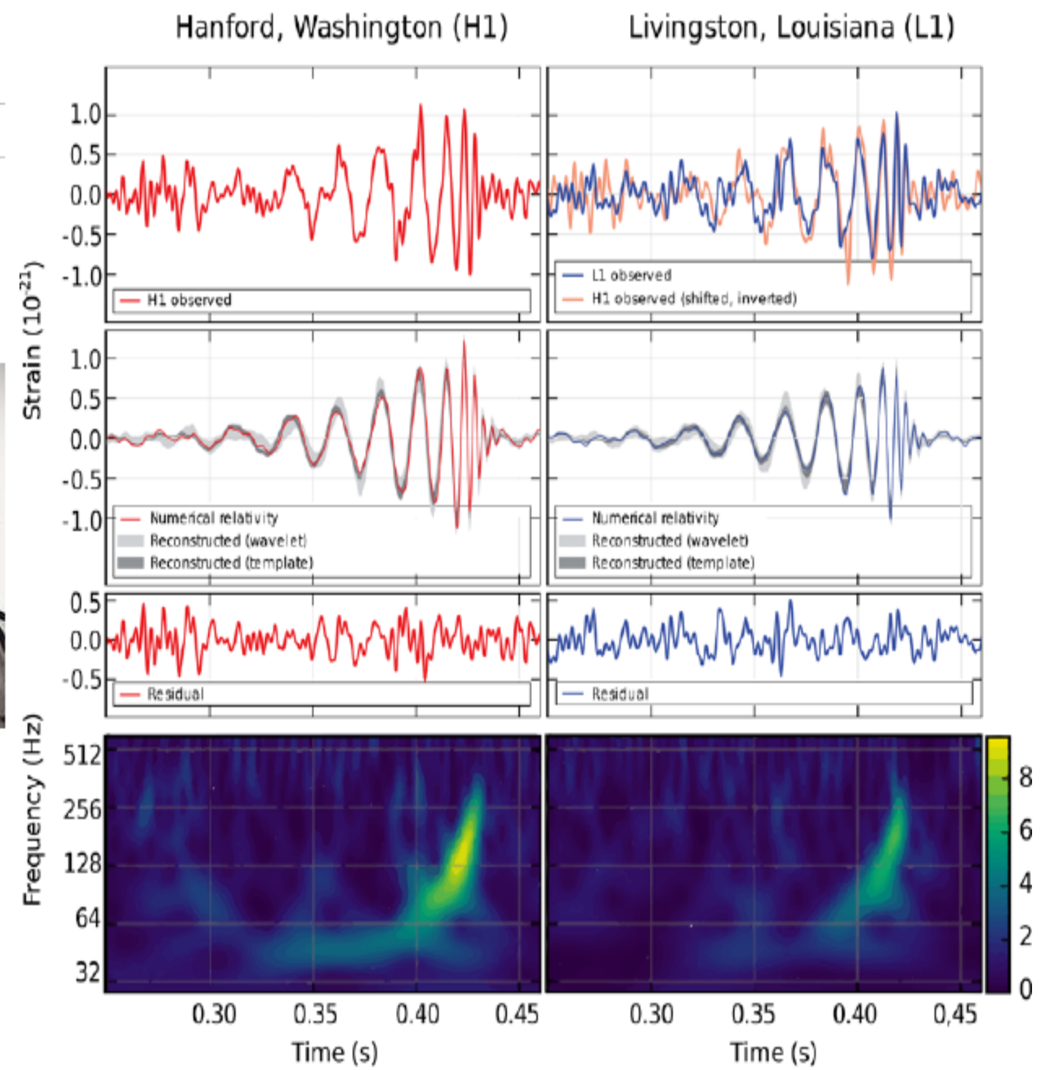
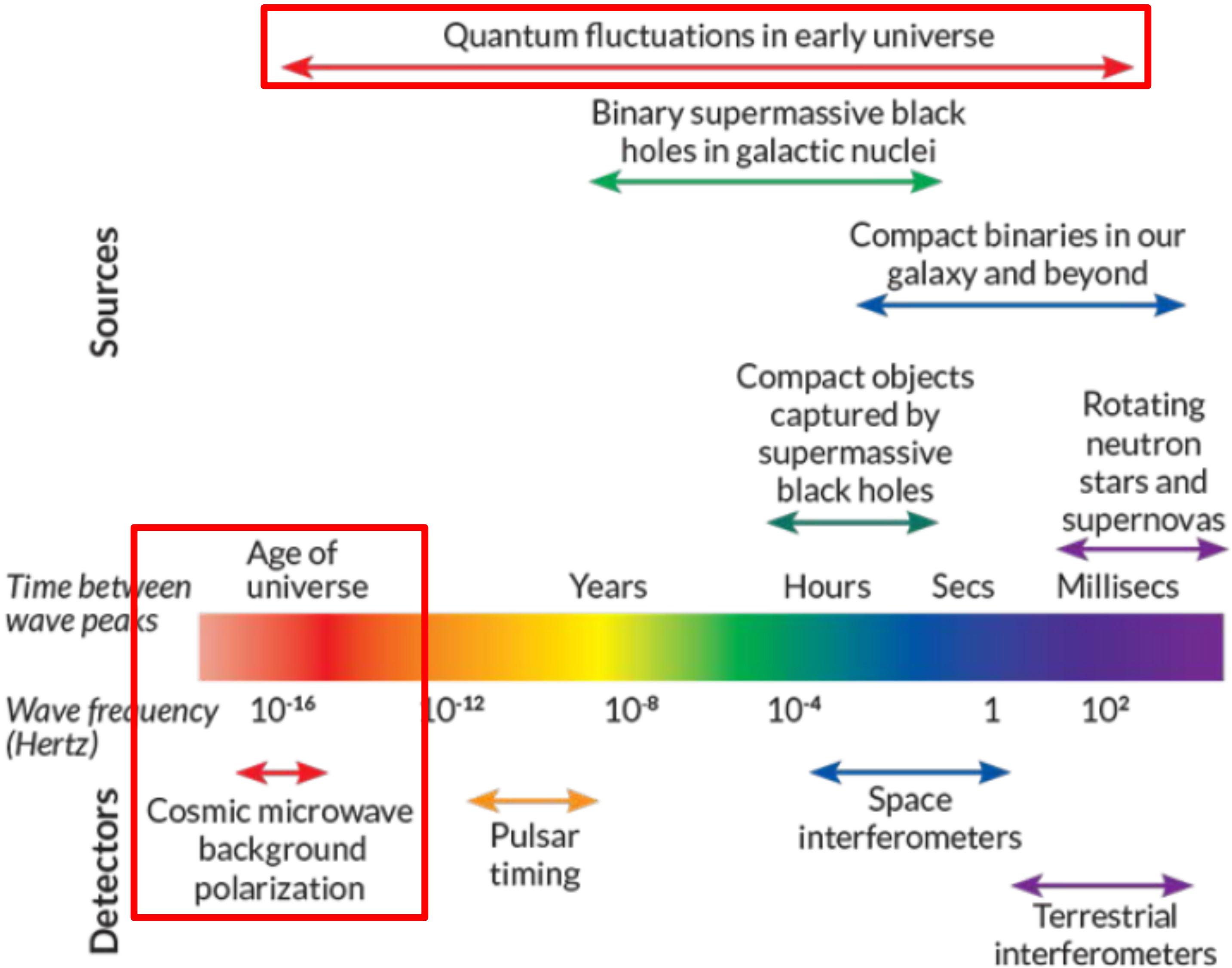
Kip S. Thorne
Prize share: 1/4

The Nobel Prize in Physics 2017 was divided, one half awarded to Rainer Weiss, the other half jointly to Barry C. Barish and Kip S. Thorne *"for decisive contributions to the LIGO detector and the observation of gravitational waves"*.



The next story: **Primordial GWs**



The Theory of Primordial Gravitational Waves

Classical Tensor Perturbations

- Consider a spatially flat Friedmann-Robertson-Walker background:

  - In cosmic time

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

  - In conformal time

$$ds^2 = a^2(\tau)(-d\tau^2 + \delta_{ij}dx^i dx^j)$$

- Tensor perturbations are **traceless** & **transverse** :

$$\delta_{ij} \to \delta_{ij} + h_{ij} \qquad\qquad h_{ij} = h_{ji} \ ; \ h_{ii} = 0 \ ; \ h_{ij,j} = 0$$

- Linearized Einstein equations (synchronous gauge)

Anisotropic stress

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} - \nabla^2 h_{ij} = 16\pi G a^2 \boxed{(T^i_j - P\delta_{ij})}$$

- Classical evolution is similar to the massless scalar field case, despite of the spacetime index.

Classical Tensor Perturbations

- Fourier expansion:

$$h_{ij}(x) = \sqrt{16\pi G} \sum_r \int \frac{d^3k}{(2\pi)^3} \epsilon_{ij}^r(\mathbf{k}) h_{\mathbf{k}}^r(\tau) e^{i\mathbf{k}\cdot\mathbf{x}}$$

where the symmetric polarization tensor is traceless & transverse, and is normalized as

$$\sum_{ij} \epsilon_{ij}^r(\mathbf{k}) \epsilon_{ij}^s(\mathbf{k})^* = 2\delta^{rs}$$

- Equation of motion in Fourier space (no source term):

$$h_{\mathbf{k}}^{r\,\prime\prime} + 2\frac{a'}{a} h_{\mathbf{k}}^{r\,\prime} + k^2 h_{\mathbf{k}}^r = 0$$

- Assuming a power law expansion:

$$a \propto \tau^n \quad n = \frac{2}{1+3w} \quad w = P/\rho$$

namely, w=1/3 for radiation; w=0 for dust matter; w~-1 for inflation

Classical Tensor Perturbations

- Equation of motion with a constant background equation of state

$$h_{\mathbf{k}}^{r\,\prime\prime} + 2\frac{n}{\tau}h_{\mathbf{k}}^{r\,\prime} + k^2 h_{\mathbf{k}}^{r} = 0$$

- General solutions in terms of Bessel functions:

$$h_k(\tau) = \tau^{1-n}\left(Aj_{\nu-1/2}(k\tau) + By_{\nu-1/2}(k\tau)\right)$$

$$\nu^2 = n(n-1) + \frac{1}{4}$$

- A little mathematical properties:

For large z:

$$J_\alpha(z) = \sqrt{\frac{2}{\pi z}}\left(\cos\left(z - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right) + e^{|\operatorname{Im}(z)|}O(|z|^{-1})\right)$$

$$Y_\alpha(z) = \sqrt{\frac{2}{\pi z}}\left(\sin\left(z - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right) + e^{|\operatorname{Im}(z)|}O(|z|^{-1})\right)$$

For small & negative z:

$$J_\alpha(z) \sim \frac{(-1)^\alpha}{(-\alpha)!}\left(\frac{2}{z}\right)^\alpha$$

$$Y_\alpha(z) \sim -\frac{(-1)^\alpha\Gamma(-\alpha)}{\pi}\left(\frac{z}{2}\right)^\alpha$$

- Thus, general solutions have two asymptotical forms:
  - When |k\tau| >> 1, oscillatory behaviors
  - When |k\tau| << 1, constant and time-varying modes

Quantization and Power Spectrum

- In the linear theory, one can take the analogy with the massless scalar field case to build the quantum theory associated with the tensor modes in a curved spacetime.

$$h_{ij}(\mathbf{x},\tau) = \sum_r \sqrt{16\pi G} \int \frac{d^3k}{(2\pi)^3} \left[\epsilon_{ij}^r(\mathbf{k}) \underbrace{h_k(\tau) a_{\mathbf{k}}^r}_{\text{physical time-dependent operator}} e^{i\mathbf{k}\cdot\mathbf{x}} + \epsilon_{ij}^r(\mathbf{k})^* h_k(\tau)^* a_{\mathbf{k}}^{r\dagger} e^{-i\mathbf{k}\cdot\mathbf{x}} \right]$$

- The conjugate momenta are expressed as

$$\pi_{ij} = \frac{\delta^{(2)} S}{\delta h'_{ij}} = \frac{1}{2} a^2 h'_{ij}$$

- Quantized Poisson brackets

$$\begin{aligned} [h_{ij}(\tau,\mathbf{k}), h_{kl}(\tau,\mathbf{k}')] &= 0\,, \\ [p_{ij}(\tau,\mathbf{k}), p_{kl}(\tau,\mathbf{k}')] &= 0\,, \\ [h_{ij}(\tau,\mathbf{k}), p_{kl}(\tau,\mathbf{k}')] &= \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \\ &\times\ \delta^{(3)}(\mathbf{k}-\mathbf{k}')\,. \end{aligned}$$

Wronskian normalization condition

$$\Rightarrow \quad \boxed{h_k \dot{h}_k^* - h_k^* \dot{h}_k = \frac{i}{a^2}}$$

Quantization and Power Spectrum

- Power spectrum is defined by

$$\langle 0|h_{ij}(\mathbf{x},\tau)h_{ij}(\mathbf{y},\tau)|0\rangle \equiv \int \frac{d^3k}{(2\pi)^3}\frac{2\pi^2}{k^3}\Delta_T^2(k,\tau)e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}$$

- Or, it is simply expressed as

$$\boxed{\Delta_T^2(k,\tau) = 64\pi G\frac{k^3}{2\pi^2}|h_k(\tau)|^2}$$

- Energy spectrum is associated with the energy density via:

$$T^{\mu\nu}_{GW} = -\frac{2}{\sqrt{\bar{g}}}\frac{\delta S_{GW}}{\delta \bar{g}_{\mu\nu}} \qquad \rho_{GW} = T_{GW}{}^0{}_0 = \bar{g}_{00}T^{00}_{GW}$$

$$\Omega_{GW}(k,\tau) \equiv \frac{1}{\rho_c(\tau)}\frac{d\langle 0|\rho_{GW}|0\rangle}{d(\ln k)} = \frac{8\pi G}{3H(\tau)^2}\frac{k^3}{2\pi^2 a^2(\tau)}\left(|\dot{h}_k(\tau)|^2 + k^2|h_k|^2\right)$$

Quantization and Power Spectrum

- Comments:
  - Primordial GWs could be originated from quantum fluctuations of the spacetime of the baby universe such that they can be accommodated with the quantum theory;
  - These quantum mechanically generated spacetime ripples need to be squeezed into classical perturbations so that the power and energy spectra can be probed observationally;
  - The above two theoretical expectations happen to be in agreement with two asymptotical solutions for primordial GWs in forms of the Bessel functions
- Question: Is there any causal mechanism that can connect two asymptotical solutions of primordial GWs dynamically?

Inflationary Cosmology

Consider a Lagrangian of a free massive scalar field:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$$

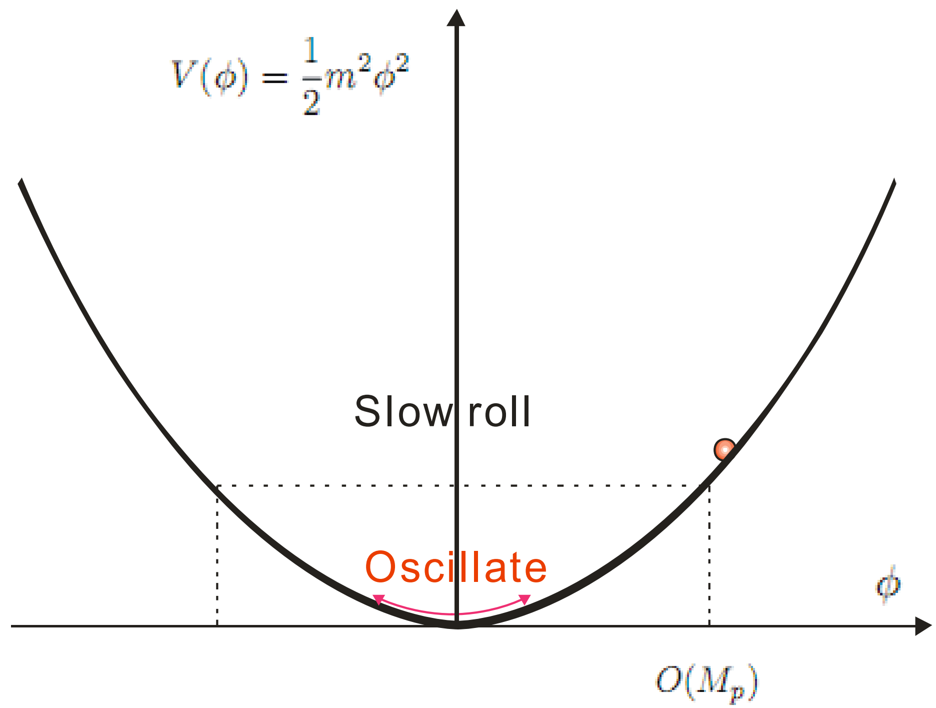
Its cosmological evolution follows the Friedmann equation and the Klein-Gordon equation:

$$H^2 = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2\right)$$

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

This model yields an accelerating phase at high energy scale, when the amplitude of the scalar is larger than the Planck mass.

Inflation



Guth '81; Linde '82;
Albrecht & Steinhardt, '82;
Starobinsky '80; Sato '81; Fang '81; …

Inflationary Cosmology

- The horizon, flatness, monopole problems can be solved

- Primordial fluctuations can lead to the formation of LSS

- Nearly scale-invariant power spectrum of primordial density fluctuations

- Small amount of primordial non-Gaussianities

- Sizable tensor modes

- Trans-Planckian perturbations?

- Initial big bang singularity?

PGWs of Inflationary Cosmology

- Slow roll inflation yields a phase of nearly exponential expansion (\tau < 0):

$$a \propto e^{Ht} \sim -\frac{1}{H\tau} \quad \Rightarrow \quad \frac{a''}{a} = \frac{2}{\tau}$$

- Slow roll parameters are introduced by

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{M_p^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \qquad \eta \equiv M_p^2 \frac{V_{,\phi\phi}}{V}$$

  and are much less than unity.

- Mukhanov-Sasaki variable and the EoM:

$$v_k \equiv h_{\mathbf{k}}/a \qquad v_k'' + (k^2 - \frac{a''}{a})v_k = 0$$

- Applying the vacuum initial condition, the solution for PGWs during inflation is given by

$$\boxed{h_k(\tau) = -\frac{H}{\sqrt{2k}}\tau\left(1 - \frac{i}{k\tau}\right)e^{-ik\tau} = H\sqrt{\frac{k}{2}}\tau^2 h_1^{(2)}(k\tau)}$$

PGWs of Inflationary Cosmology

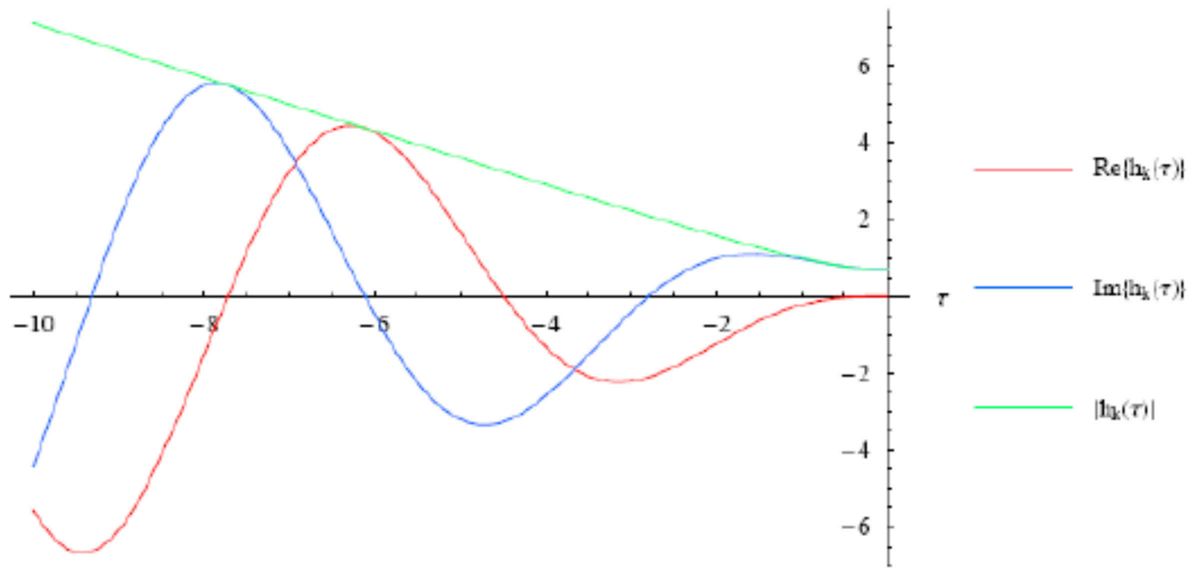
- When |k\tau|>>1, the sub-Hubble solution connects with the vacuum fluctuations

$$h_{\mathbf{k}}^{\mathrm{UV}} \to -\frac{H\tau}{\sqrt{2k}} e^{-ik\tau}$$

- When |k\tau|<<1, the super-Hubble solution get frozen to become constant mode

$$h_{\mathbf{k}}^{\mathrm{IR}} \to \frac{iH}{\sqrt{2k^3}}$$

- Hubble crossing condition: k = aH or |k\tau| = 1
- For a k=1 mode, the solution is numerically given in the following



PGWs of Inflationary Cosmology

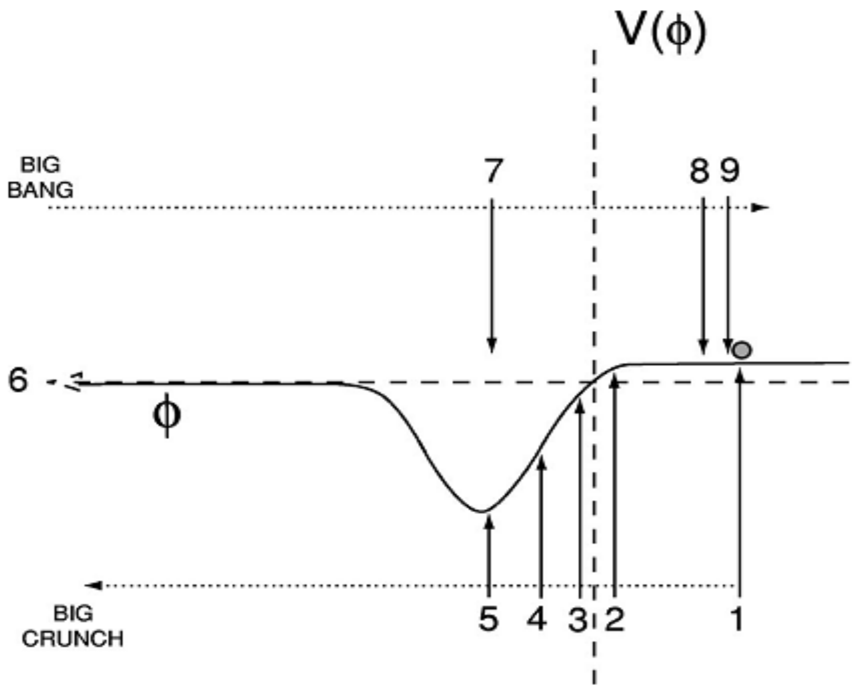
- At the end of inflation (epsilon=1), for the tensor modes outside the Hubble radius:
  - Power spectrum: $$P_T \equiv \Delta_T^2(k, \tau_e) = \frac{2H_*^2}{\pi^2 M_p^2}$$
  - Spectral index: $$n_T \equiv \frac{d \ln P_T}{d \ln k} = -2\epsilon_*$$
  - Tensor-to-Scalar ratio: $$r \equiv \frac{P_T}{P_\zeta} = 16\epsilon_* \quad \Rightarrow \quad \boxed{r = -8n_T}$$

    Consistency relation for the model of single field slow roll inflation
  - Energy spectrum: $$\Omega_{GW}(k, \tau_e) = \frac{H^2}{6\pi^2 M_p^2} = \frac{1}{12}\Delta_T^2(k, \tau_e)$$ $$\Omega_{GW}(k, \tau_{\text{today}}) = T(k; \tau_{\text{today}}, \tau_e) \times \Omega_{GW}(k, \tau_e)$$
- Today's energy spectrum can be related to the primordial energy spectrum via the transfer function.

Ekpyrotic Bounce

The collision of two M branes in 5D gives rise to a nonsingular cyclic universe, and the description of EFT in 4D is

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} \mathcal{R} + \frac{1}{2} (\partial\phi)^2 - V(\phi) + \beta^4(\phi)(\rho_M + \rho_R) \right)$$



1 DE domination
2 decelerated expansion
3 turnaround
4 ekpyrotic contracting phase
5 before big crunch
6 a singular bounce in 4D
7 after big bang
8 radiation domination
9 matter domination

Khoury, Ovrut, Steinhardt & Turok, PRD, '01

Ekpyrotic Bounce

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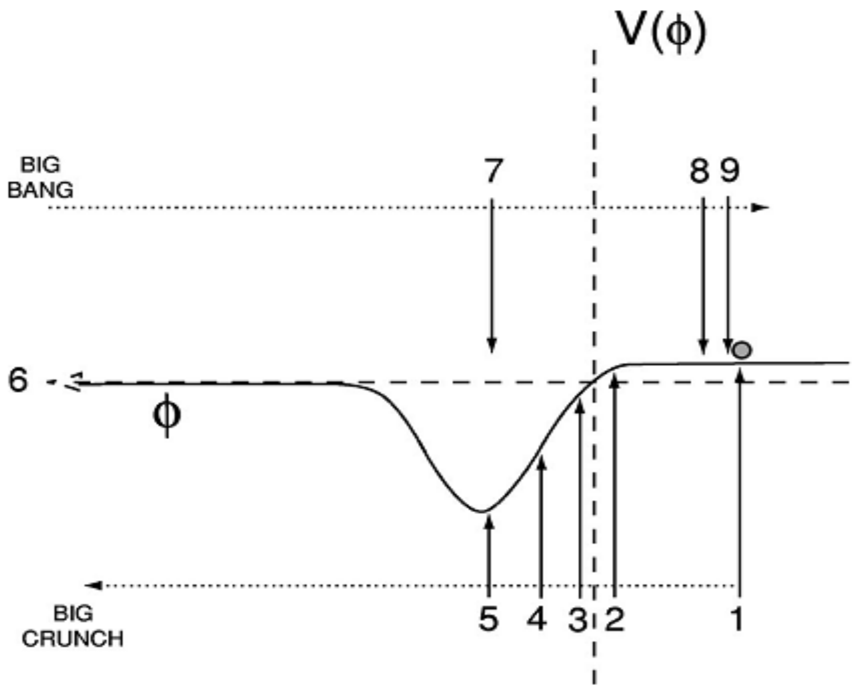


1 DE domination
2 decelerated expansion
3 turnaround
4 ekpyrotic contracting phase
5 before big crunch
6 a **singular** bounce in 4D
7 after big bang
8 radiation domination
9 matter domination

Failure of effective field theory description, uncertainty involved in perturbations.

Ekpyrotic Bounce

Predictions by Ekpyrotic model:

- Nearly scale-invariant power spectrum
- Large primordial non-Gaussianities
- Almost no tensor modes

Nonsingular Bounce

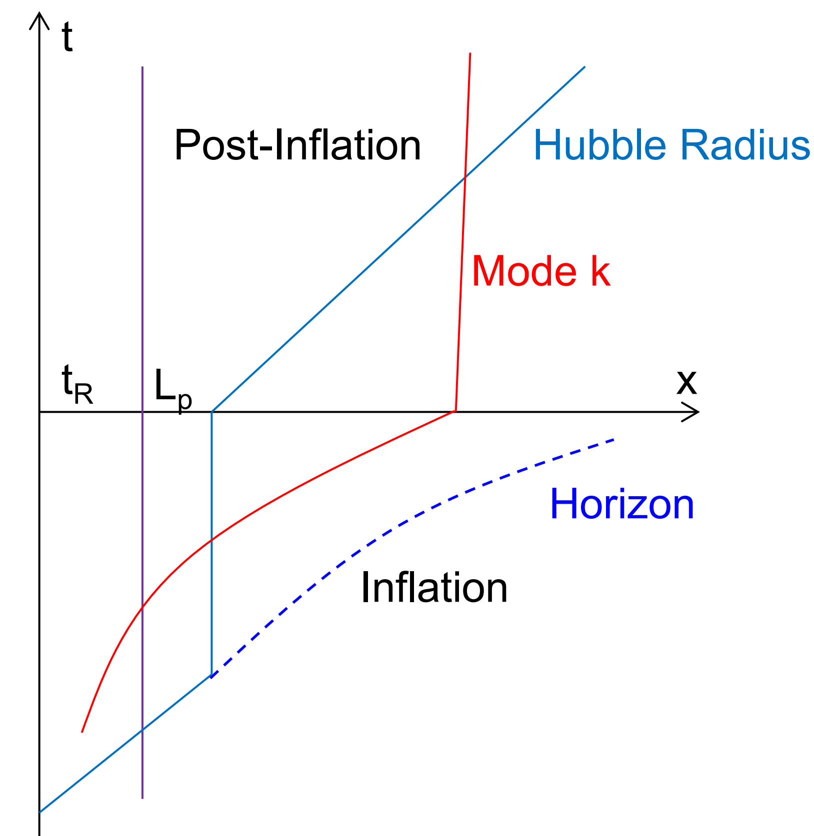
- Pre-Big-Bang
  (non-perturbative effects) Gasperini & Veneziano ’92
- String Gas Cosmology
  (thermal non-local system) Brandenberger & Vafa ’89
- *Matter Bounce* Cosmology
  (NEC violating matter. Wands ’98; Finelli & Brandenberger ’01)
- Loop Quantum Cosmology
  (quantum structure of spacetime. Ashtekar 0812.0177)
- Mirage Cosmology
  (nonconventional braneworld. Kehagias & Kiritsis ’99)
- Lee-Wick Bounce
  (high-order derivatives. *CYF*, Qiu, Brandenberger & Zhang ’08)
- New Ekpyrotic Cosmology
  (ghost condensate. Buchbinder, Khoury & Ovrut ’07; Creminelli, Luty, Nicolis & Senatore ’06)
- Galilean Genesis
  (galilean field. Creminelli, Nicolis & Trincherini ’10)
- …

Sketch Plot

Crucial facts:

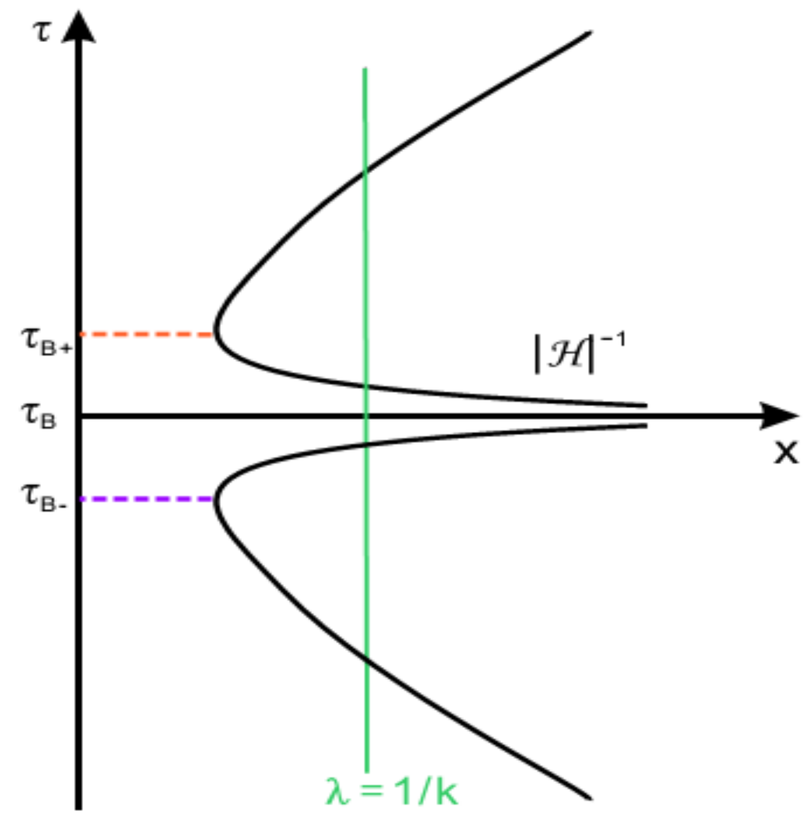
- Fluctuations originate on sub-Hubble scales
- Fluctuations propagate for a long time on super-Hubble scales
- Trans-Planckian problem: Inflation; Bounce

Brandenberger, hep-ph/9910410



Inflationary prediction:
a scale-invariant spectrum

CYF, Qiu, Brandenberger & Zhang, 0810.4677



Can we obtain a scale-invariant spectrum as in inflation?

PGWs of Matter Bounce Cosmology

- Matter bounce requires a phase of matter-dominated contraction (w=0, \tau < 0):

$$a \sim \tau^2 \quad \Rightarrow \quad \frac{a''}{a} = \frac{2}{\tau}$$

- Similar to the slow roll, there is

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3}{2}(1+w)$$

- Inserting to the EoM of Mukhanov-Sasaki variable

$$v_k \equiv h_{\mathbf{k}}/a \qquad v_k'' + (k^2 - \frac{a''}{a})v_k = 0$$

- Using the vacuum initial condition, the solution for PGWs in matter contraction takes

$$\begin{aligned} v = (\eta - \tilde{\eta}_{B-})^{1/2}\{&A_k^T J_{-(3/2)}[k(\eta - \tilde{\eta}_{B-})] \\ &+ B_k^T J_{3/2}[k(\eta - \tilde{\eta}_{B-})]\}, \end{aligned}$$

$$A_k^T = i\frac{\sqrt{\pi}}{2} \quad \text{and} \quad B_k^T = -\frac{\sqrt{\pi}}{2}$$

PGWs of Matter Bounce Cosmology

- Therefore, the asymptotic form of the solution in matter contracting phase is

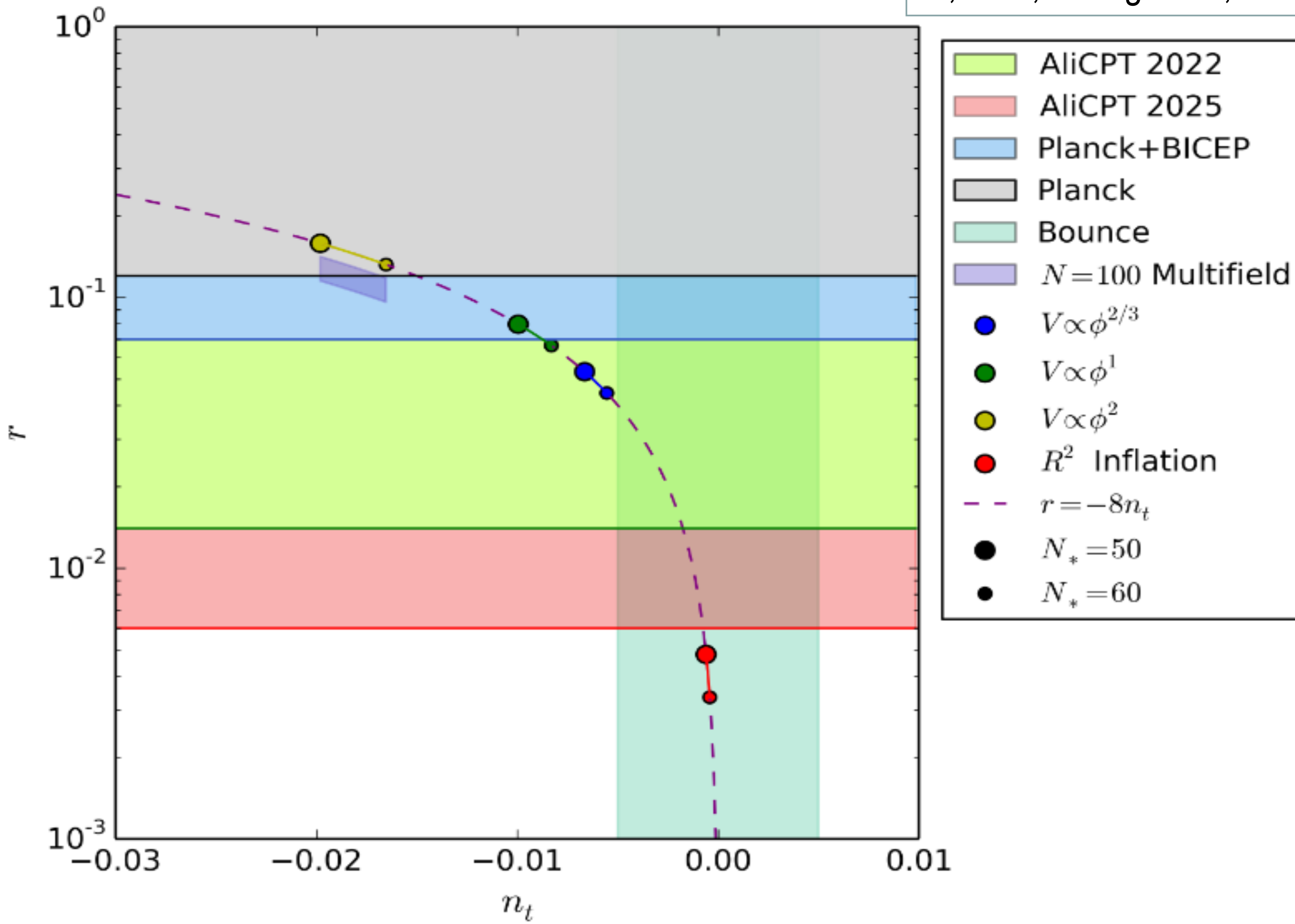
$$v(k,\eta) = \begin{cases} -\frac{i}{\sqrt{2}}k^{-(3/2)}(\eta - \tilde{\eta}_{B-})^{-1}, & \text{outside Hubble radius} \\ \frac{1}{\sqrt{2k}}e^{-ik(\eta-\tilde{\eta}_{B-})}, & \text{inside Hubble radius} \end{cases}$$

- After evolving PGWs through the bouncing phase to connect with thermal expansion,
  - Power spectrum: $P_T(k) = G\frac{32k^3}{\pi}\left|\frac{v^f}{a}\right|^2 = \frac{2\rho_{B+}}{27\pi^2 M_p^4}$
  - Spectral index: $n_T \equiv \frac{d\ln P_T}{d\ln k} = 0 \pm (\text{bg error})$
  - Tensor-to-Scalar ratio: $r \equiv \frac{P_T}{P_\zeta} \lesssim O(1)$

Consistency relation is broken for matter bounce

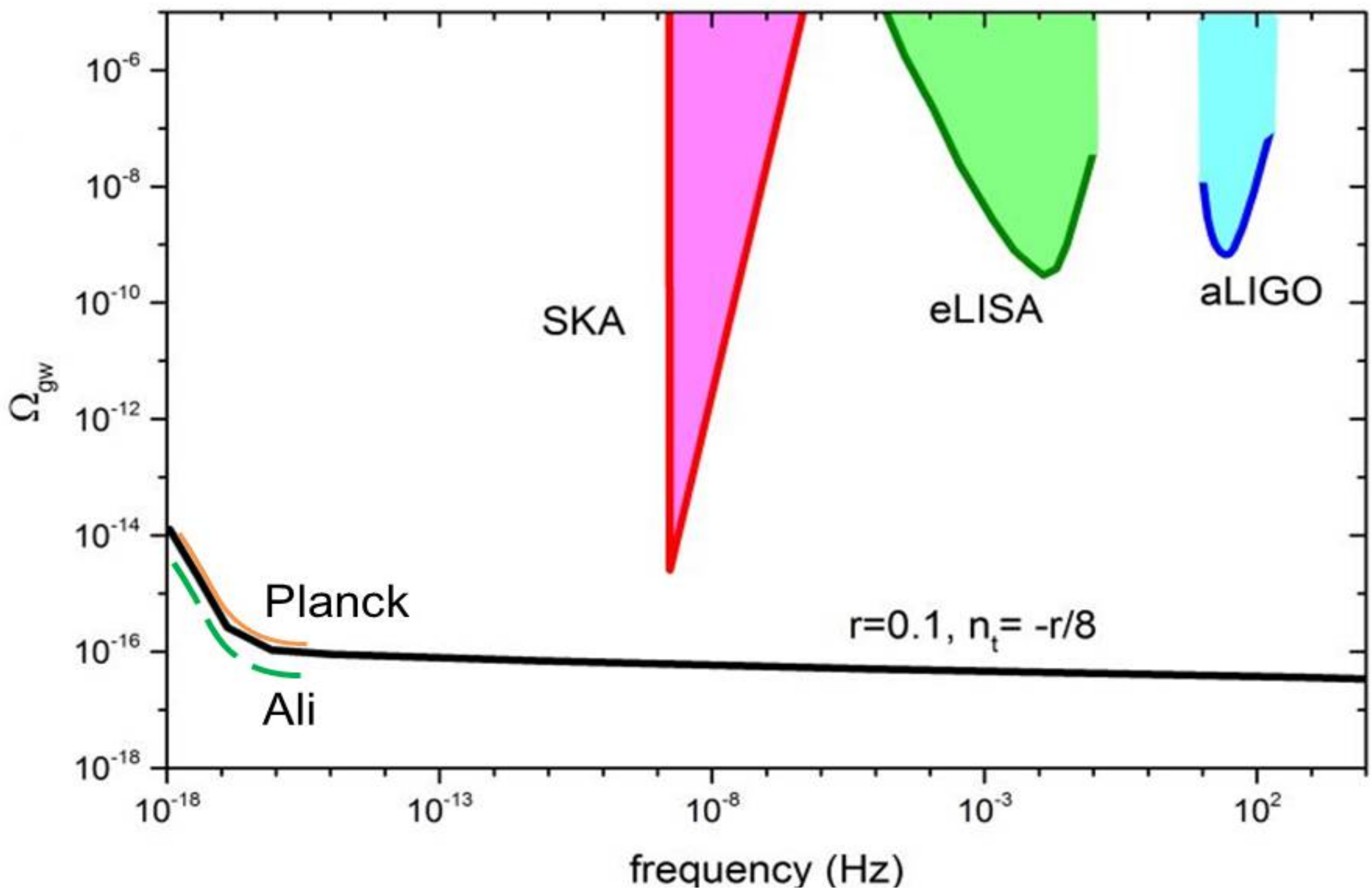
Consistency relation of inflationary PGWs and comparison with matter bounce and experiments

CYF & Zhang, 1605.01840;
Li, *CYF*, Zhang et.al., 1710.03047



Energy spectrum of inflationary PGWs and comparison with experimental sensitivities

CYF & Zhang, 1605.01840



Evolutions after primordial era

- Based on the big bang theory, assuming a CDM model:

  Radiation + Matter + instantaneous transition

$$a(\tau) = \begin{cases} H_0\sqrt{\Omega_{r0}}\tau, & 0 \leq \tau \leq \tau_{eq} \\ a_{eq}\left(\frac{\tau}{\tau_{eq}}\right)^2, & \tau_{eq} < \tau \leq \tau_0 \end{cases}$$

- Solutions:

  - Radiation expansion

$$h_k(\tau) = A j_0(k\tau) + B y_0(k\tau) \longrightarrow A = h_k(0), \;\; B = 0$$

  - Matter expansion

$$h_k(\tau) = A_k\left(\frac{3j_1(k\tau)}{k\tau}\right) + B_k\left(\frac{3y_1(k\tau)}{k\tau}\right)$$

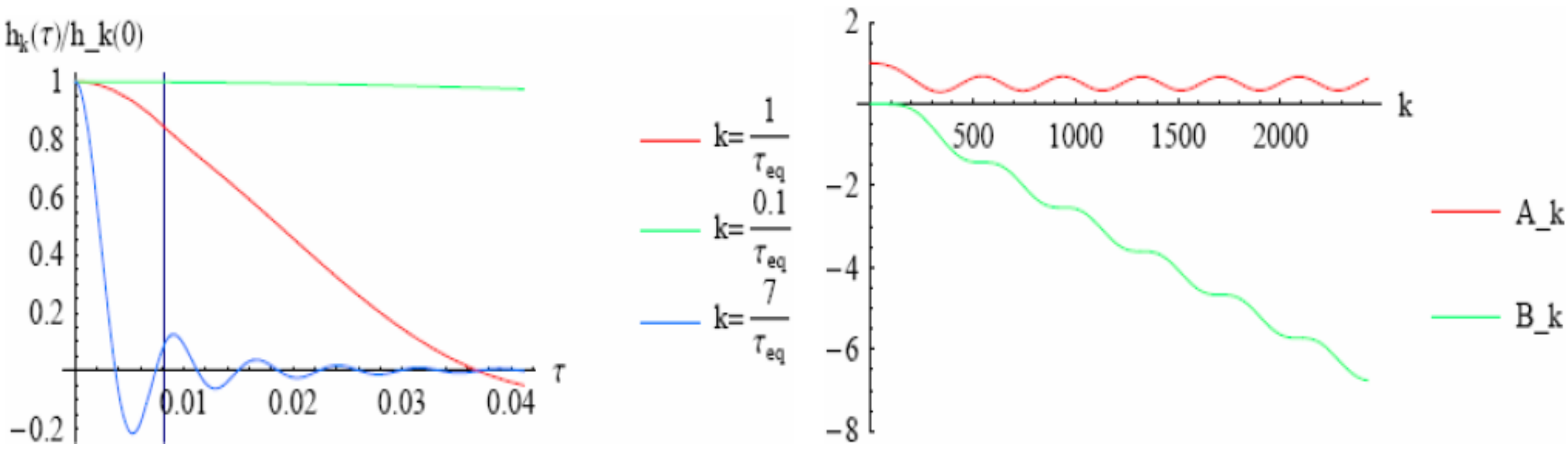
Evolutions after primordial era

- Continuity of the solution yields the coefficients:

$$A_k = h_k(0)\frac{3k\tau_{eq} - k\tau_{eq}\cos(2k\tau_{eq}) + 2\sin(2k\tau_{eq})}{6k\tau_{eq}}$$

$$B_k = h_k(0)\frac{2 - 2k^2\tau_{eq}^2 - 2\cos(2k\tau_{eq}) - k\tau_{eq}\sin(2k\tau_{eq})}{6k\tau_{eq}}$$

- Modes reenter the Hubble radius during radiation and matter phases:



- The transfer coefficients would be more complicated if more elements are taken into account, namely, the dark energy era, smooth transitions, anisotropic stress (cosmic neutrinos), etc.

Evolutions after primordial era

- Comments:
  - The generation of PGWs are associated with the environment of the primordial phase;
  - Evolutions after the primordial era, i.e. transfer functions, are associated with phases of radiation, matter, dark energy, and phase transitions that the universe has experienced, as well as the relativistic d.o.f., such as free-streaming neutrinos;
  - The study of the cosmic background of PGWs can reveal important information about the evolution of the universe throughout the whole history.
- Question: How can we probe them in cosmological observations?

From PGWs to the CMB

CMB Blackbody background

- CMB is a (nearly) perfect blackbody characterized by a phase space distribution function

$$f = \frac{1}{e^{E/T} - 1}$$

where the temperature $T(x, \hat{n}, t)$ is observed at our position $x=0$ and time t_0 to be nearly isotropic with a mean temperature of 2.725K

- Our observable is the temperature anisotropy

$$\Theta(\hat{\mathbf{n}}) \equiv \frac{T(0, \hat{\mathbf{n}}, t_0) - \bar{T}}{\bar{T}}$$

- Given that physical processes essentially put a band limit on this function it is useful to decompose it into a complete set of harmonic coefficients

PGWs induce temperature fluctuations & polarization in CMB

- Consider that the universe is filled with CMB photons that do not scatter. Thus, the photon energies are affected only by the metric.

- As a toy model, consider a single monochromatic plane-wave GW, which appears as a tensor perturbation to the FRW metric:

$$ds^2 = a^2(\eta)\left[d\eta^2 - dx^2(1+h_+) + dy^2(1-h_+) + dz^2\right]$$

$$h_+(x,\eta) \simeq h(\eta)e^{ik\eta}e^{-ikz}$$

  which describes a linearly-polarized plane wave along the z axis.

- Photons that propagate freely through this spacetime experience a frequency shift $d\nu$ in an expansion interval determined by the geodesic equation, which takes the form:

$$\frac{1}{\nu}\frac{d\nu}{d\eta} = -\frac{1}{2}(1-\mu^2)\cos 2\phi e^{-ikz}\frac{d}{d\eta}(he^{ik\eta})$$

  where μ is the cosine of the angle that photon trajectory makes with z axis, ϕ is the azimuthal angle of the photon's trajectory.

PGWs induce temperature fluctuations & polarization in CMB

- The above effect is polarization-independent, but polarization is induced by Thomson scattering of this anisotropic radiation field. To account for the polarization, we must follow the time evolution of four distribution functions:

$$f_s(\boldsymbol{x}, \boldsymbol{q}; \eta)$$

q is photon momentum; s=(I,Q,U,V) are four Stokes parameters

- At unperturbed background:

$$\bar{f}_I(\boldsymbol{q}, \boldsymbol{x}; \eta) = \left[e^{h\nu/k_B T(\eta)} - 1\right]^{-1} \qquad \bar{f}_Q = \bar{f}_U = \bar{f}_V = 0$$

- Then we introduce the perturbations

$$\Delta_s e^{i\boldsymbol{k}\cdot\boldsymbol{x}} = 4\delta f_s/(\partial \bar{f}/\partial \ln T)$$

$$\Delta_I = \tilde{\Delta}_I (1-\mu)^2 \cos 2\phi, \qquad \Delta_Q = \tilde{\Delta}_Q (1+\mu)^2 \cos 2\phi, \qquad \Delta_U = \tilde{\Delta}_U 2\mu \sin 2\phi,$$

which are variables as functions only of μ and time.

Harmonic Analysis for CMB Polarizations

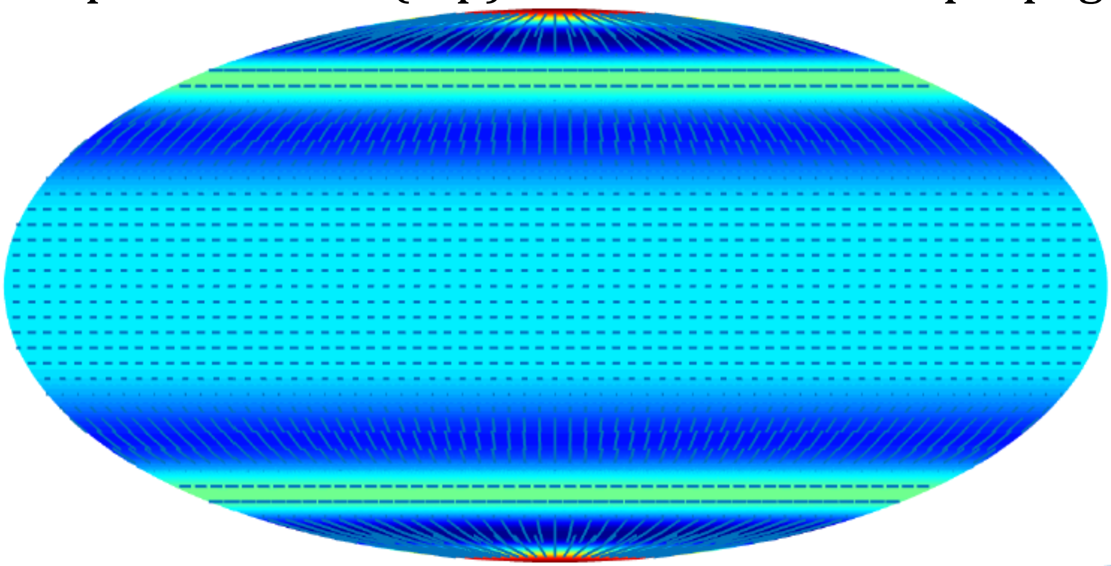
- Comments:
  - Thomson scattering induces no circular polarization, so $\Delta_V = 0$ at all times.
  - For GWs induced perturbations, there is: $\tilde{\Delta}_Q = -\tilde{\Delta}_U$
    since the orientation of the photon polarization is fixed by the direction of the photon w.r.t. the GW polarization.
  - Consequently, there are only two Boltzmann equations:
    $\dot{\tilde{\Delta}}_I + ik\mu\tilde{\Delta}_T = -\dot{h} - \dot{\kappa}\left[\tilde{\Delta}T - \Psi\right]$, $\qquad \dot{\tilde{\Delta}}_Q + ik\mu\tilde{\Delta}_Q = -\dot{\kappa}\left[\Delta_P + \Psi\right]$
    where the dot w.r.t. conformal time and Legendre moments:
    $$\tilde{\Delta}_{I\ell}(\eta) = (1/2)\int_{-1}^{1} d\mu\, \tilde{\Delta}_I(\mu;\eta) P_\ell(\mu)$$
  - The r.h.s. account for Thomson scattering.

astro-ph/9506072

Harmonic Analysis for CMB Polarizations

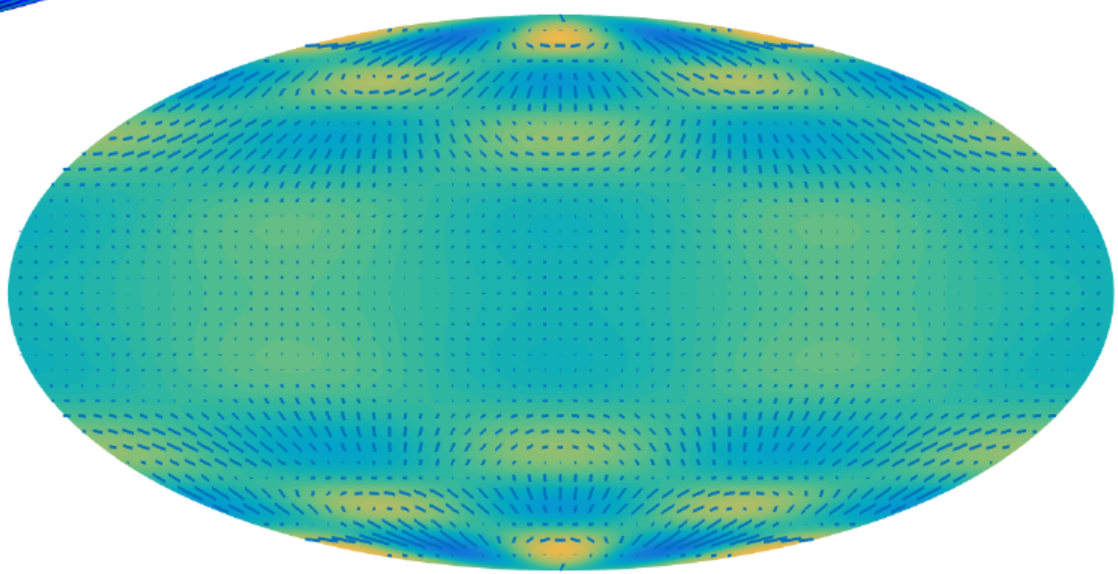
1510.06042

CMB temperature-polarization pattern induced by one Fourier mode of scalar perturbation (top) and one GW mode propagating in z direction (bottom):



Scalar mode

Tensor mode

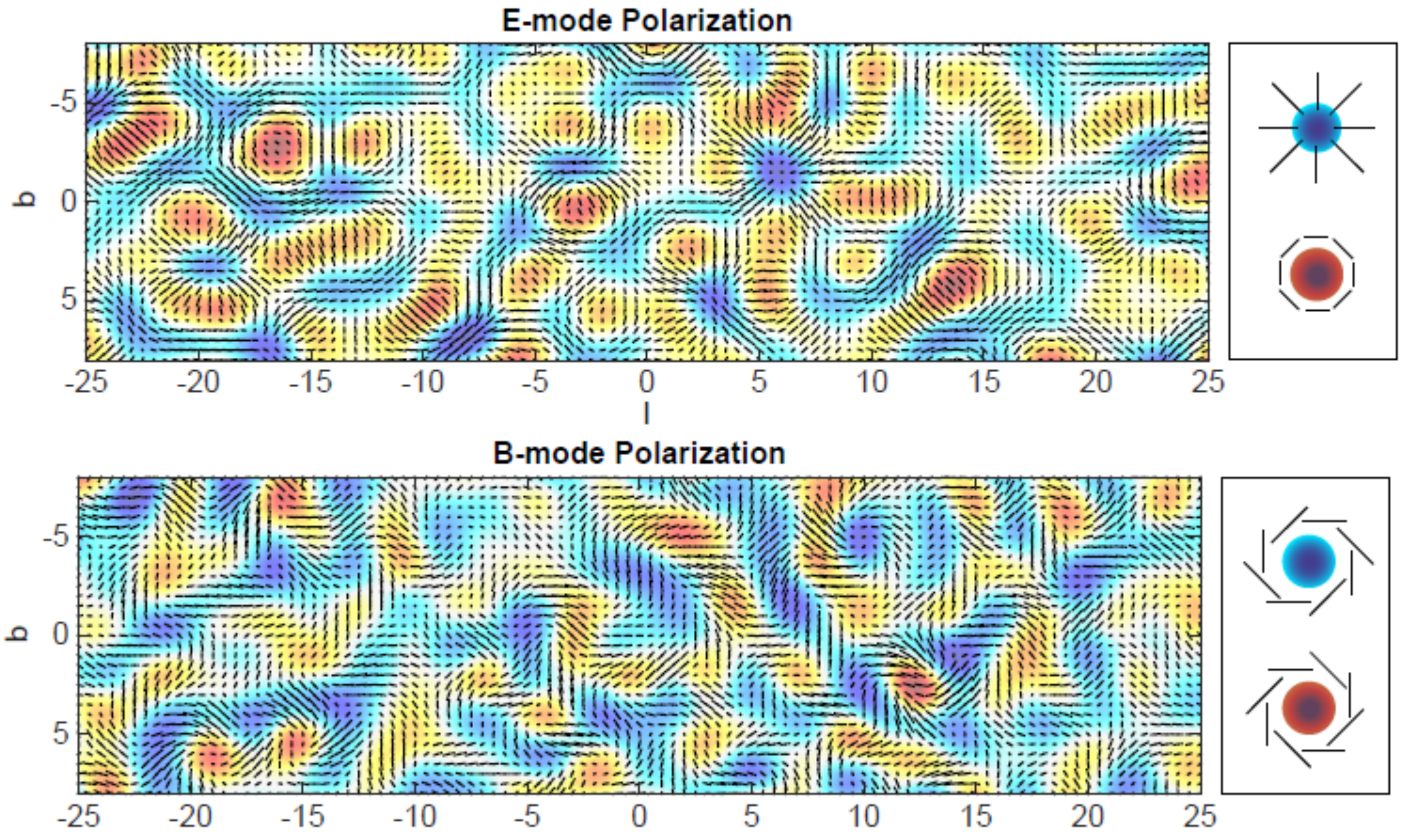


There are variations in U, the component of the polarization π/4 with respect to constant-longitude lines.
This is a signature of B mode induced by GW!

Harmonic Analysis for CMB Polarizations

- Note that the polarization is spin-2 field:

$$\begin{pmatrix} Q & U \\ U & -Q \end{pmatrix} \Rightarrow \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} Q & U \\ U & -Q \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$



E and B modes from PGWs

- PGWs of k wave-number can induce the polarization tensor:

$$\mathcal{P}^{ab}_{k,+}(\theta,\phi) = \frac{T_0}{4\sqrt{2}} \sum_\ell (2\ell+1) P_\ell(\cos\theta) \tilde{\Delta}_{Q\ell} \begin{pmatrix} (1+\cos^2\theta)\cos 2\phi & 2\cot\theta\sin 2\phi \\ 2\cot\theta\sin 2\phi & -(1+\cos^2\theta)\csc^2\theta\cos 2\phi \end{pmatrix}$$

- It yields the E & B coefficients:

$$a^{\mathrm{E}\,k,+}_{\ell m} = \frac{\sqrt{\pi(2\ell+1)}}{4(\delta_{m,2}+\delta_{m,-2})^{-1}} \left[\frac{(\ell+2)(\ell+1)\tilde{\Delta}_{Q,\ell-2}}{(2\ell-1)(2\ell+1)} + \frac{6\ell(\ell+1)\tilde{\Delta}_{Q\ell}}{(2\ell+3)(2\ell-1)} + \frac{\ell(\ell-1)\tilde{\Delta}_{Q,\ell+2}}{(2\ell+3)(2\ell+1)} \right]$$

$$a^{\mathrm{B}\,k,+}_{\ell m} = \frac{-i}{2\sqrt{2}} \sqrt{\frac{2\pi}{(2\ell+1)}} (\delta_{m,2}-\delta_{m,-2}) \left[(\ell+2)\tilde{\Delta}_{Q,\ell-1} + (\ell-1)\tilde{\Delta}_{Q,\ell+1} \right]$$

- The angular power of B mode for fixing k takes:

$$C^{\mathrm{BB},\,k,+}_\ell = \frac{1}{2l+1} \sum_m |a^B_{\ell m}|^2 = \frac{\pi}{2} \left[\frac{\ell+2}{2\ell+1} \tilde{\Delta}_{Q,\ell-1} + \frac{\ell-1}{2\ell+1} \tilde{\Delta}_{Q,\ell+1} \right]^2$$

- Integrating out k, one gets the BB angular power spectrum

$$\boxed{C^{\mathrm{BB}}_\ell = \frac{1}{2\pi} \int k^2\, dk \left[\frac{\ell+2}{2\ell+1} \tilde{\Delta}_{Q,\ell-1}(k) + \frac{\ell-1}{2\ell+1} \tilde{\Delta}_{Q,\ell-1}(k) \right]^2}$$

E and B modes from PGWs

- Comments:
  - Similar process applies for C_l^{EE}
  - The E-B cross correlation vanishes for the standard model
  - Taking the same analysis, it is easy to see that scalar perturbation only produces T and E, simply due to the fact that density perturbations do not produce a curl at linear level
  - But, B modes may still arise from density perturbations at nonlinear order
  - Question: How large are these foreground contaminations?

Lensing induced B modes

- The most relevant nonlinear effect is weak gravitational lensing induced by (scalar type) density perturbations between us and the CMB surface of last scatter.

astro-ph/9803150

- The Stokes parameters displace along a given direction:

$$\begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{\text{obs.}} (\theta) = \begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{\text{ls}} (\theta + \delta\theta) \simeq \begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{\text{ls}} (\theta) + \delta\theta \cdot \nabla \begin{pmatrix} T \\ Q \\ U \end{pmatrix}_{\text{ls}} (\theta)$$

where $\delta\theta = \nabla\Phi$ is the lensing deflection along gravitational potential.

- If no PGWs, there is only E mode at LSS with:

$$\tilde{Q}(\ell) = 2\tilde{E}(\ell)\cos 2\varphi_\ell \qquad U(\ell) = -2E(\ell)\sin 2\varphi_\ell$$

Lensing induced B modes

- Gravitational deflection leads to

$$B(\ell) = \frac{1}{2}[\sin 2\varphi_\ell \, \delta Q(\ell) - \cos 2\varphi_\ell \, \delta U(\ell)] = \int \frac{d^2 \boldsymbol{l_1}}{(2\pi)^2} [\boldsymbol{\ell}_1 \cdot (\boldsymbol{\ell} - \boldsymbol{\ell}_1)] E(\boldsymbol{\ell}_1) \Phi(\boldsymbol{\ell} - \boldsymbol{\ell}_1) \sin 2\varphi_{\boldsymbol{\ell}_1}$$

- The angular power spectrum of lensing B modes takes

$$\boxed{C_\ell^{\mathrm{BB}} = \int \frac{d^2 \boldsymbol{l_1}}{(2\pi)^2} [\boldsymbol{\ell}_1 \cdot (\boldsymbol{\ell} - \boldsymbol{\ell}_1)]^2 \sin^2 2\varphi_{\boldsymbol{\ell}_1} C^{\Phi\Phi}_{|\boldsymbol{\ell} - \boldsymbol{\ell}_1|} C^{\mathrm{EE}}_{\boldsymbol{\ell}_1}}$$

- It was for the first time detected by the SPT in 2013.

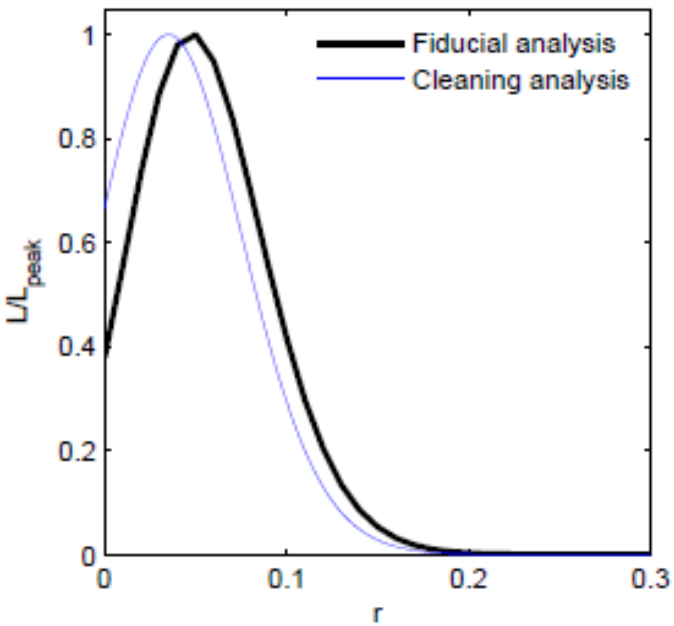
1307.5830

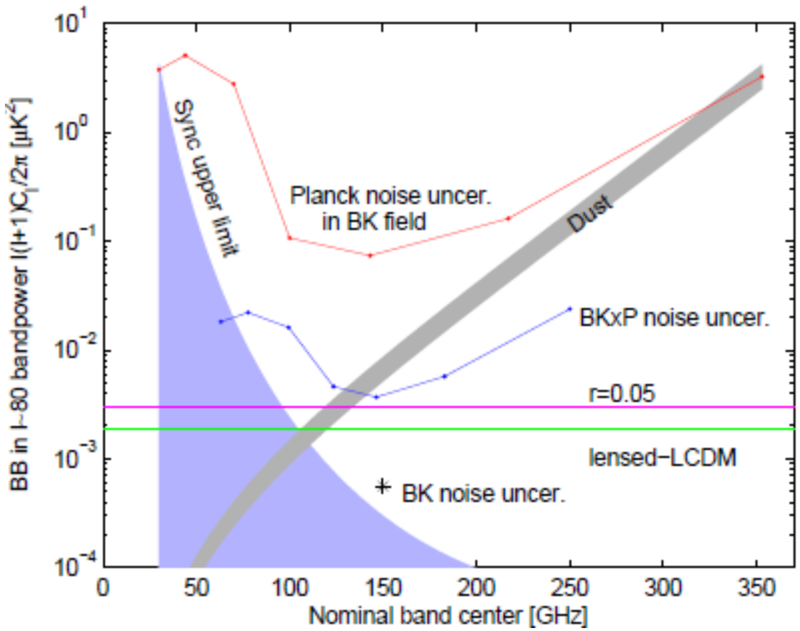
Foreground contributions to B modes

- Galactic foregrounds:
  - Synchrotron: Galactic synchrotron emission is dominant at frequencies below 100 GHz, and both WMAP and Planck have observed its polarization signature at frequencies from 30 to 90 GHz;
  - Dust: Above 100 GHz, thermal emission from asymmetric dust in the interstellar medium, which align themselves with the Galactic magnetic field, induces a strong polarization signal;
  - …
- One must use techniques of de-lensing and non-Gaussian diagnosis to eliminate foreground contaminations to extract signals of PGWs.

So far where we are …

1502.00612

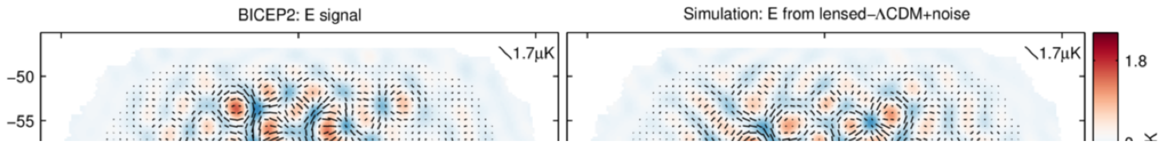


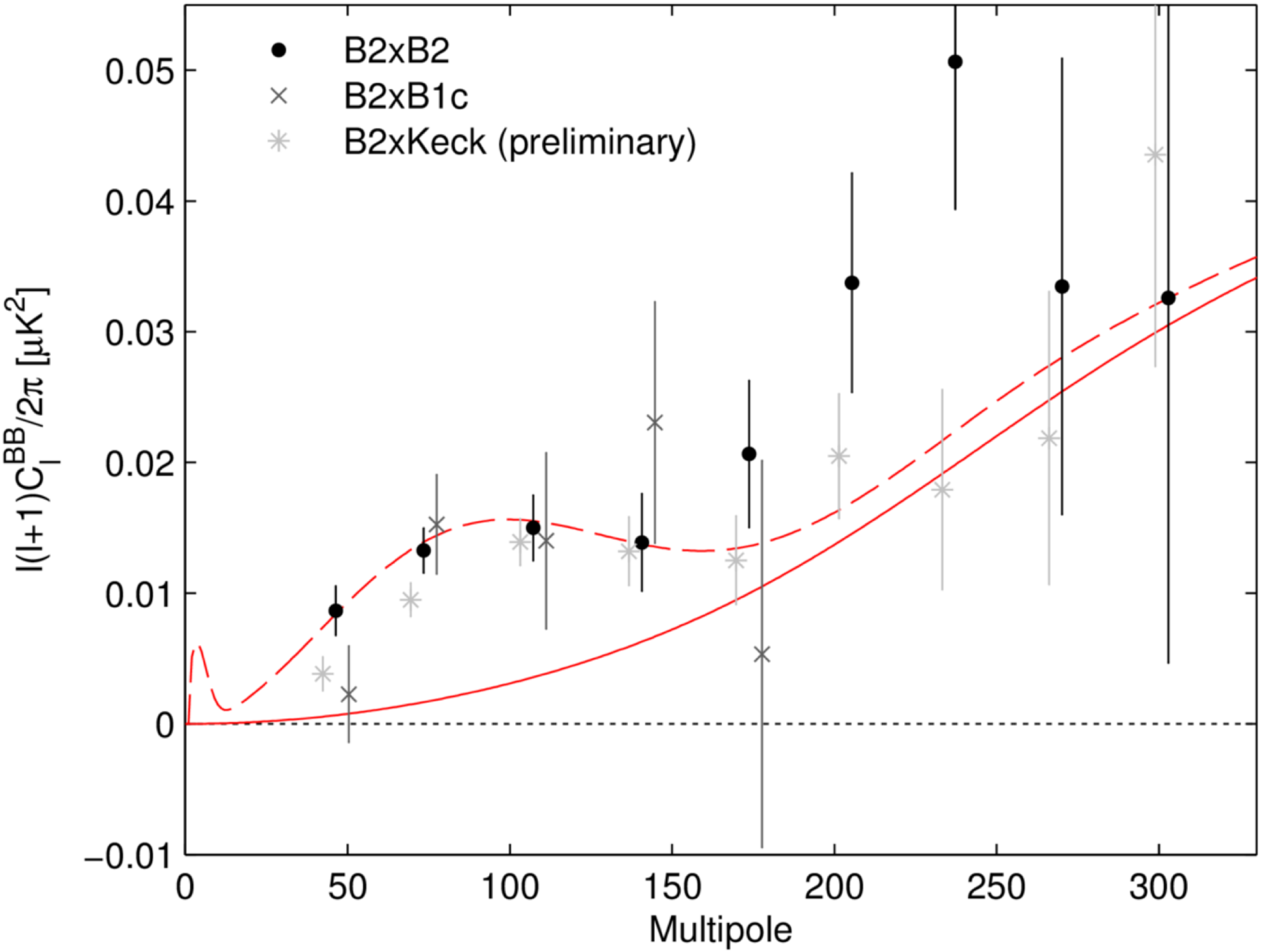


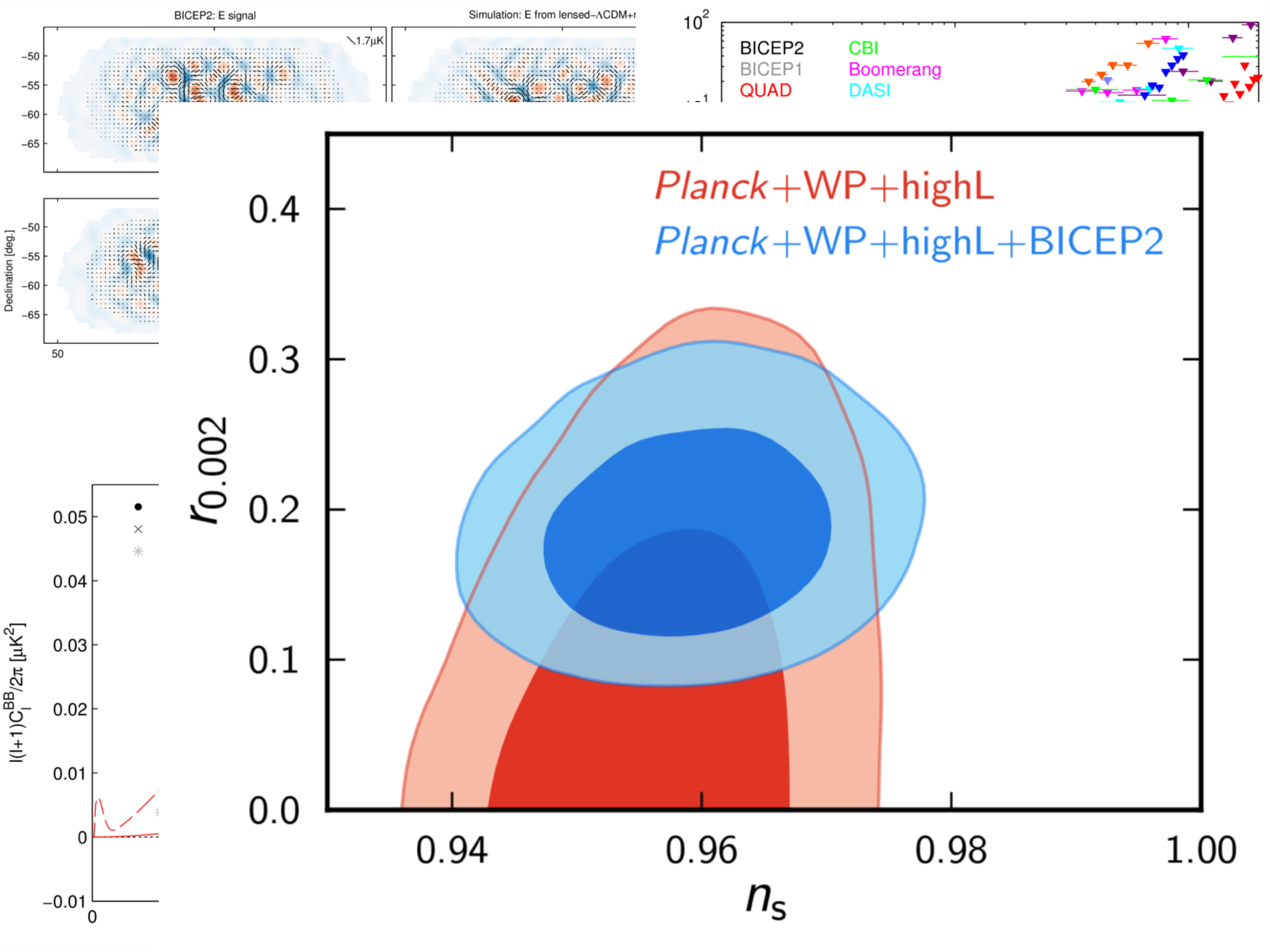
- No signal of PGWs: r < 0.07 at 2σ under a joint analysis of data from BICEP/Keck Array & Planck 2015.

A good but not perfect lesson from BICEP2

BICEP instrument at the South Pole


BICEP2: E signal
Simulation: E from lensed-ΛCDM+noise
1.7μK
1.8
−50
−55


B2xB2
B2xB1c
B2xKeck (preliminary)
0.05
0.04
0.03
0.02
0.01
0
−0.01
0
50
100
150
200
250
300
Multipole
l(l+1)C_l^BB/2π [μK^2]



BICEP2: E signal
Simulation: E from lensed-ΛCDM+r
1.7μK
Declination [deg.]
BICEP2
BICEP1
QUAD
CBI
Boomerang
DASI
l(l+1)C_l^BB/2π [μK^2]
Planck+WP+highL
Planck+WP+highL+BICEP2
$r_{0.002}$
n_s
0.0
0.1
0.2
0.3
0.4
0.94
0.96
0.98
1.00

BICEP2 gives:

$$r \equiv \frac{P_T}{P_S} = 0.20^{+0.07}_{-0.05} \quad (68\% \quad \text{C.L.})$$

Note: dust foreground contamination was not considered!

Planck13 yields:

$$\ln(10^{10} A_s) = 3.089^{+0.024}_{-0.027}$$

$$n_s = 0.9603 \pm 0.0073 \quad (68\% \quad \text{C.L.})$$

$$r < 0.11 \ (95\% \ \text{C.L.})$$

There exists a severe tension between BICEP2 and Planck !

Implications on inflation models from string cosmology

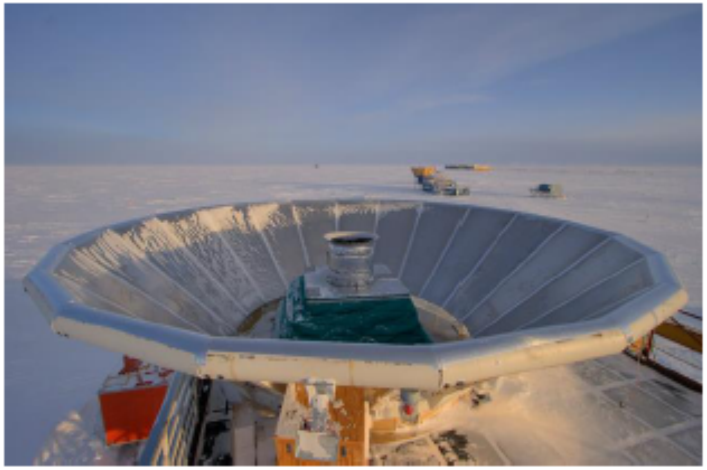
Burgess, Cicoli, Quevedo, 1306.3512

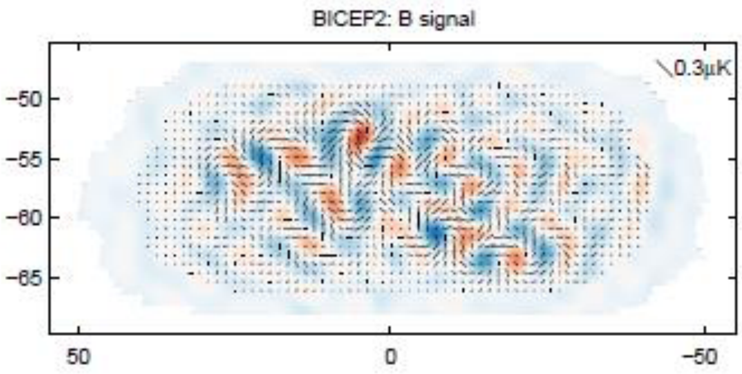
String Scenario	n_s	r
$D3/\overline{\text{D3}}$ Inflation	$0.966 \leq n_s \leq 0.972$	$r \leq 10^{-5}$
Inflection Point Inflation	$0.92 \leq n_s \leq 0.93$	$r \leq 10^{-6}$
DBI Inflation	$0.93 \leq n_s \leq 0.93$	$r \leq 10^{-7}$
Wilson Line Inflation	$0.96 \leq n_s \leq 0.97$	$r \leq 10^{-10}$
D3/D7 Inflation	$0.95 \leq n_s \leq 0.97$	$10^{-12} \leq r \leq 10^{-5}$
Racetrack Inflation	$0.95 \leq n_s \leq 0.96$	$r \leq 10^{-8}$
N-flation	$0.93 \leq n_s \leq 0.95$	$r \leq 10^{-3}$
Axion Monodromy	$0.97 \leq n_s \leq 0.98$	$0.04 \leq r \leq 0.07$
Kahler Moduli Inflation	$0.96 \leq n_s \leq 0.967$	$r \leq 10^{-10}$
Fibre Inflation	$0.965 \leq n_s \leq 0.97$	$0.0057 \leq r \leq 0.007$
Poly-instanton Inflation	$0.95 \leq n_s \leq 0.97$	$r \leq 10^{-5}$

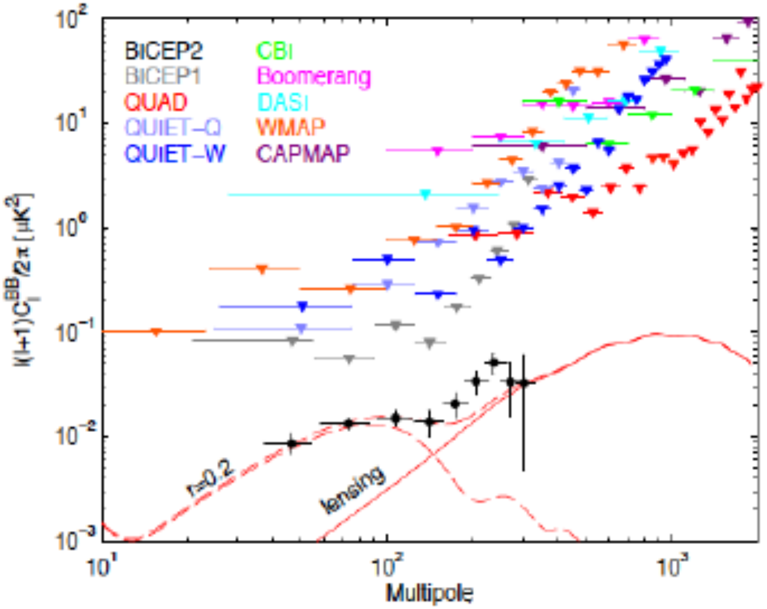
Table 1. Predictions for the cosmological observables n_s and r for different string inflationary models.

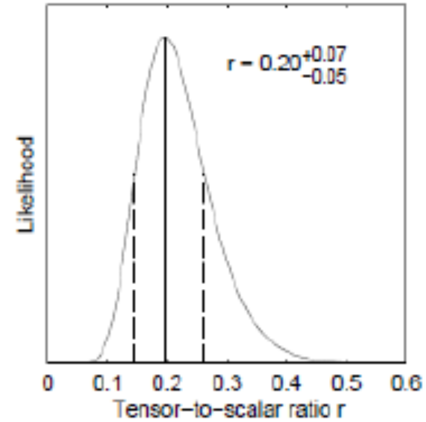
If BICEP2 were correct, all above models would have been ruled out!

BICEP2 claimed the detection of primordial GWs for the first time, but...

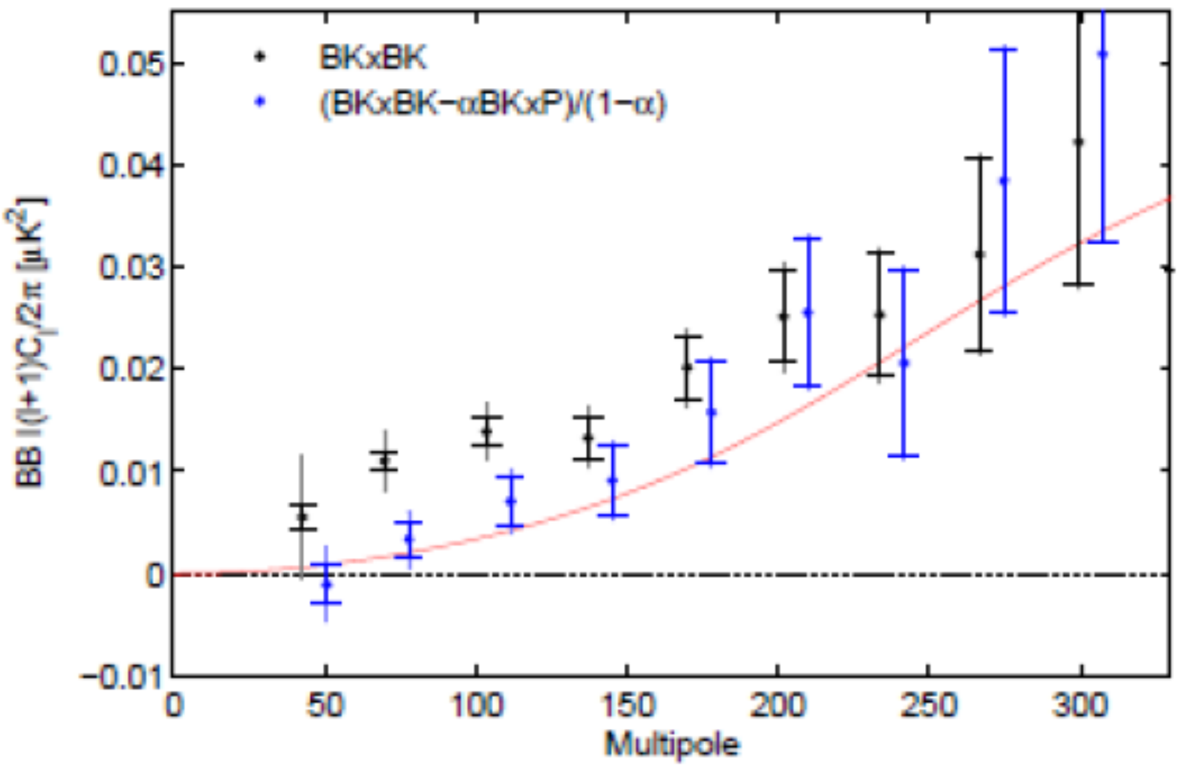




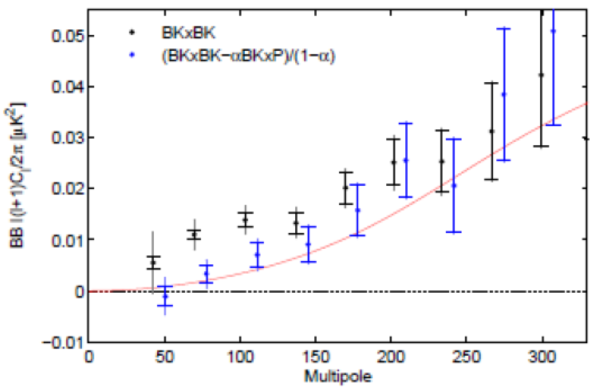




BICEP2 signals are very likely from the foreground dust contamination



BICEP2 signals are very likely from the foreground dust contamination

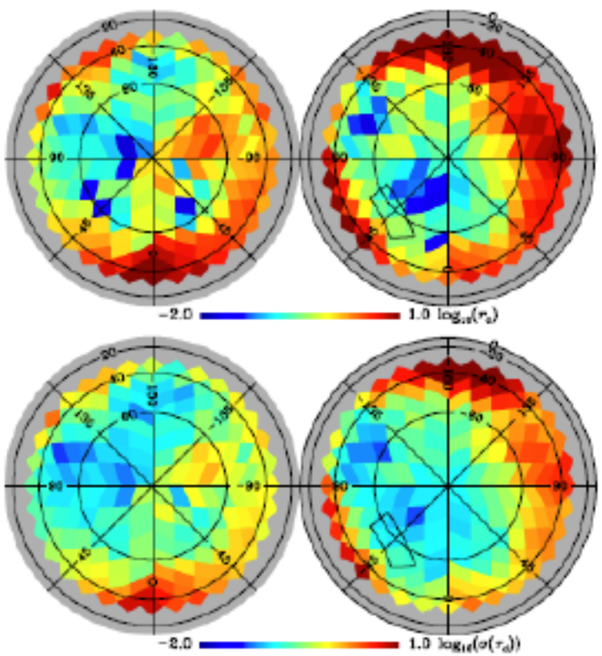


No winner yet! Who first hit to 21 points?

Searching for primordial GWs becomes important in the community.

Polarization foreground from galaxy

full sky coveraged is required !

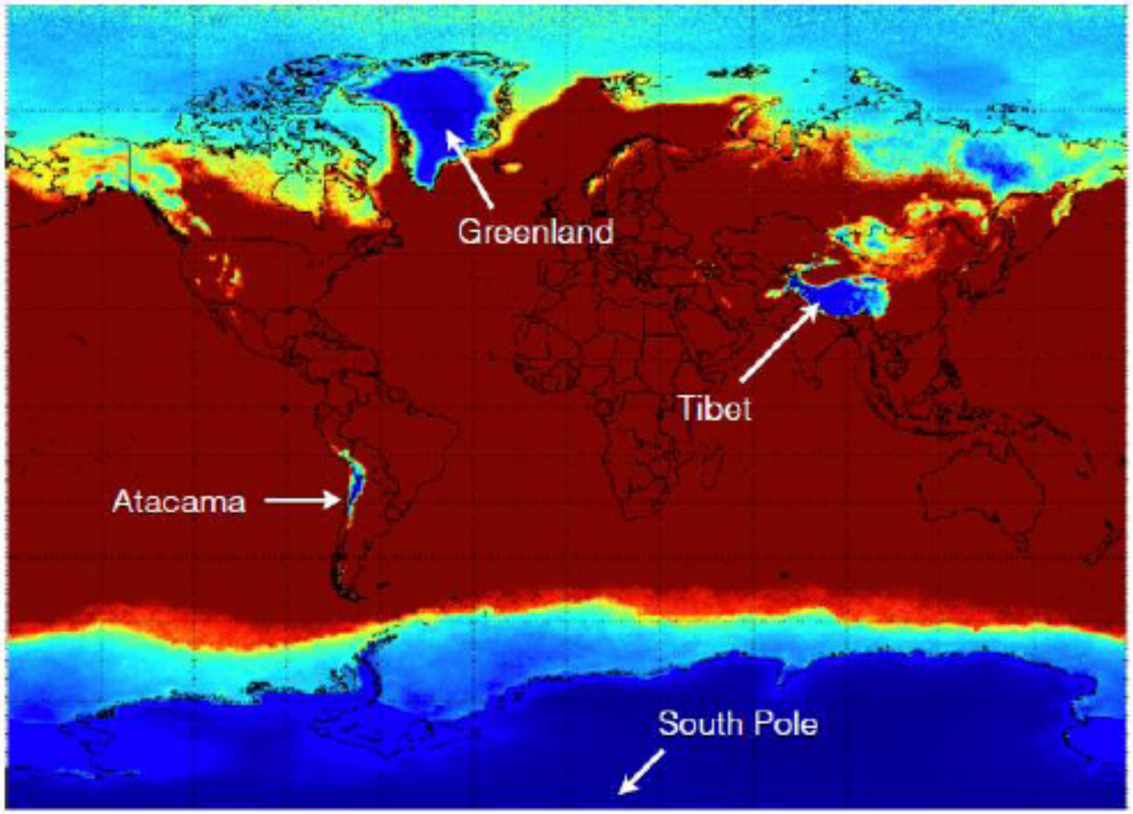


north hemisphere　　south hemisphere

- Planck can provide us the full sky coverage, but the S/N is very limited;
- After Planck, there is so far no further space-based projects;
- The ground-based CMB polarization projects will be the key developments in the next decade.

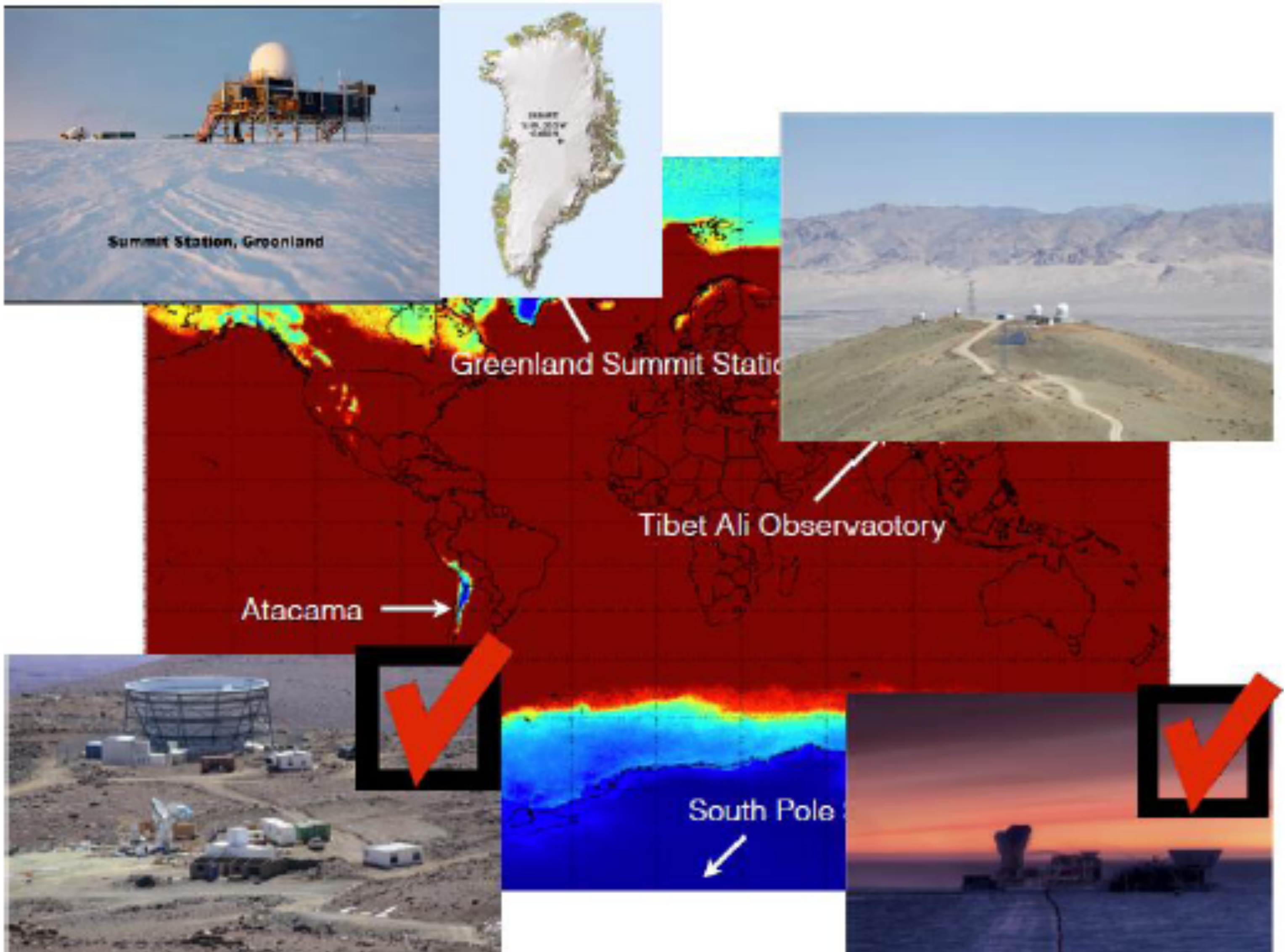
A full sky coverage is needed!

How many places suitable for CMB?

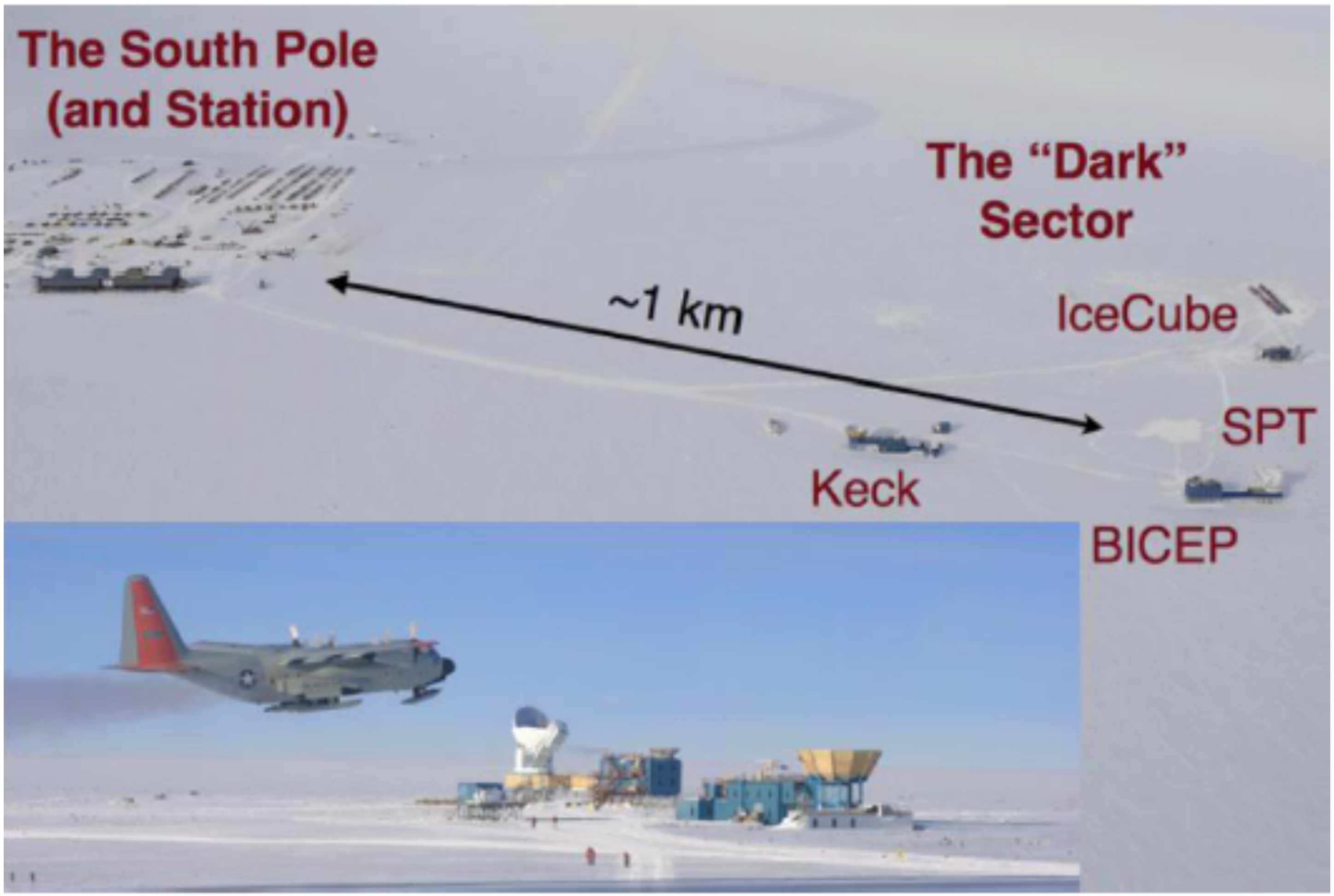


- Blue areas indicate high atmospheric transmission rate, which are suitable for CMB observations!
- Four best places on Earth: Greenland, Tibet, Atacama desert, Antarctica

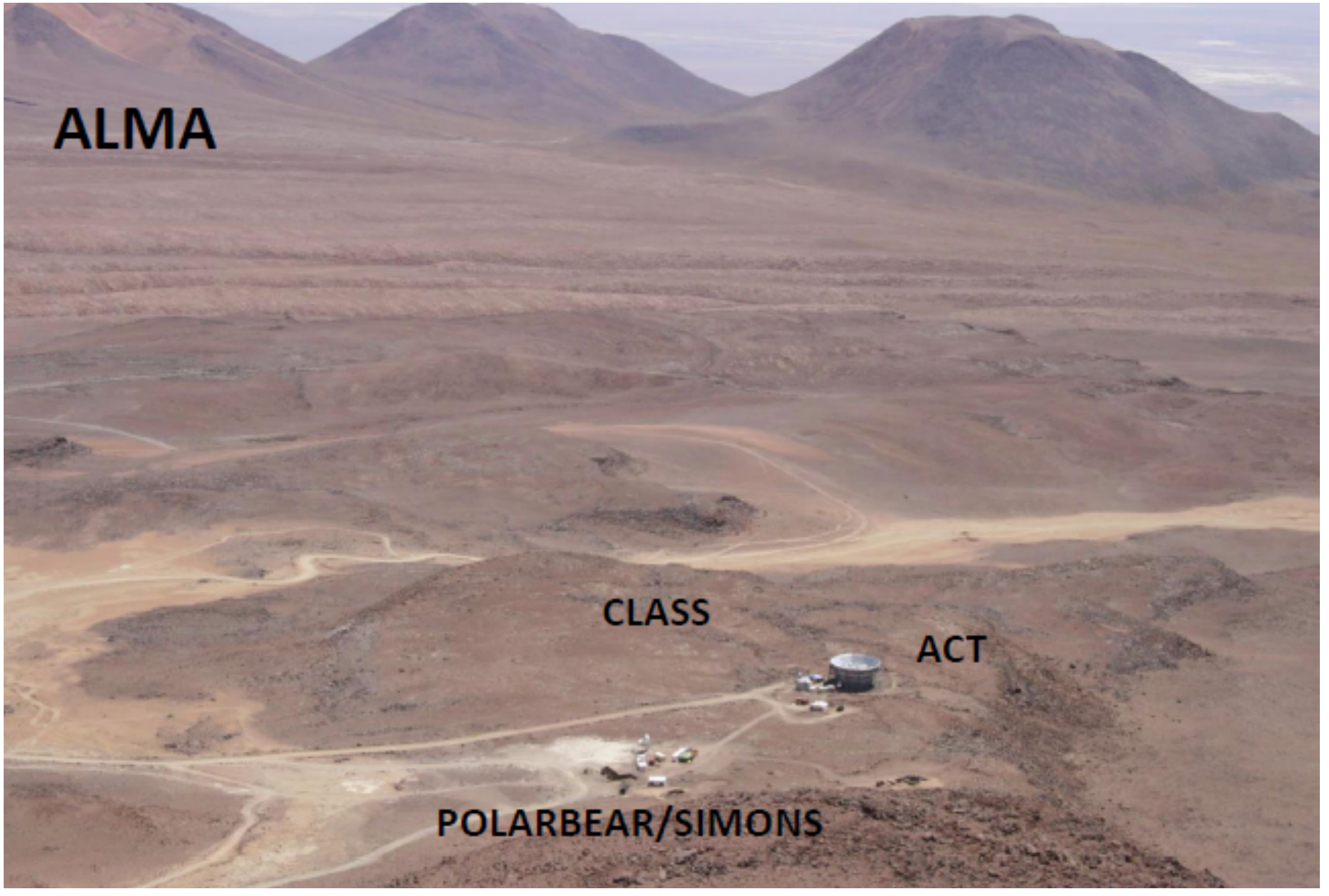
Ground-based CMB experiments



CMB at South Pole



CMB at Atacama



Full-sky coverage expects the CMB experiments in the north part of the earth

Ali (Ngari) Vs Greenland

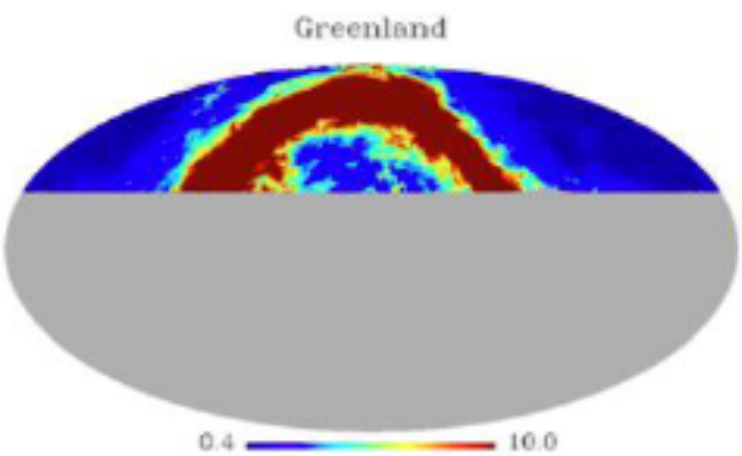
So far there is no CMB experiment in the north earth.

Facilities:

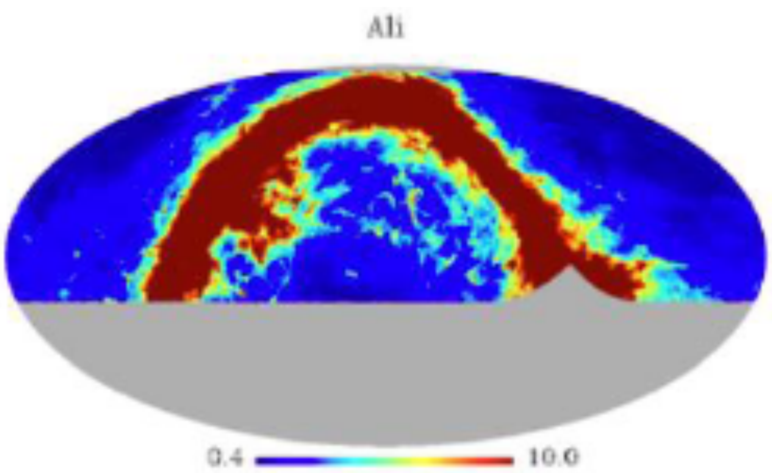
- Ali site has well established facilities including internet, electricity, public transportation, …
- Site at Greenland is now only supplied by the US army, …

Ali (Ngari) Vs Greenland

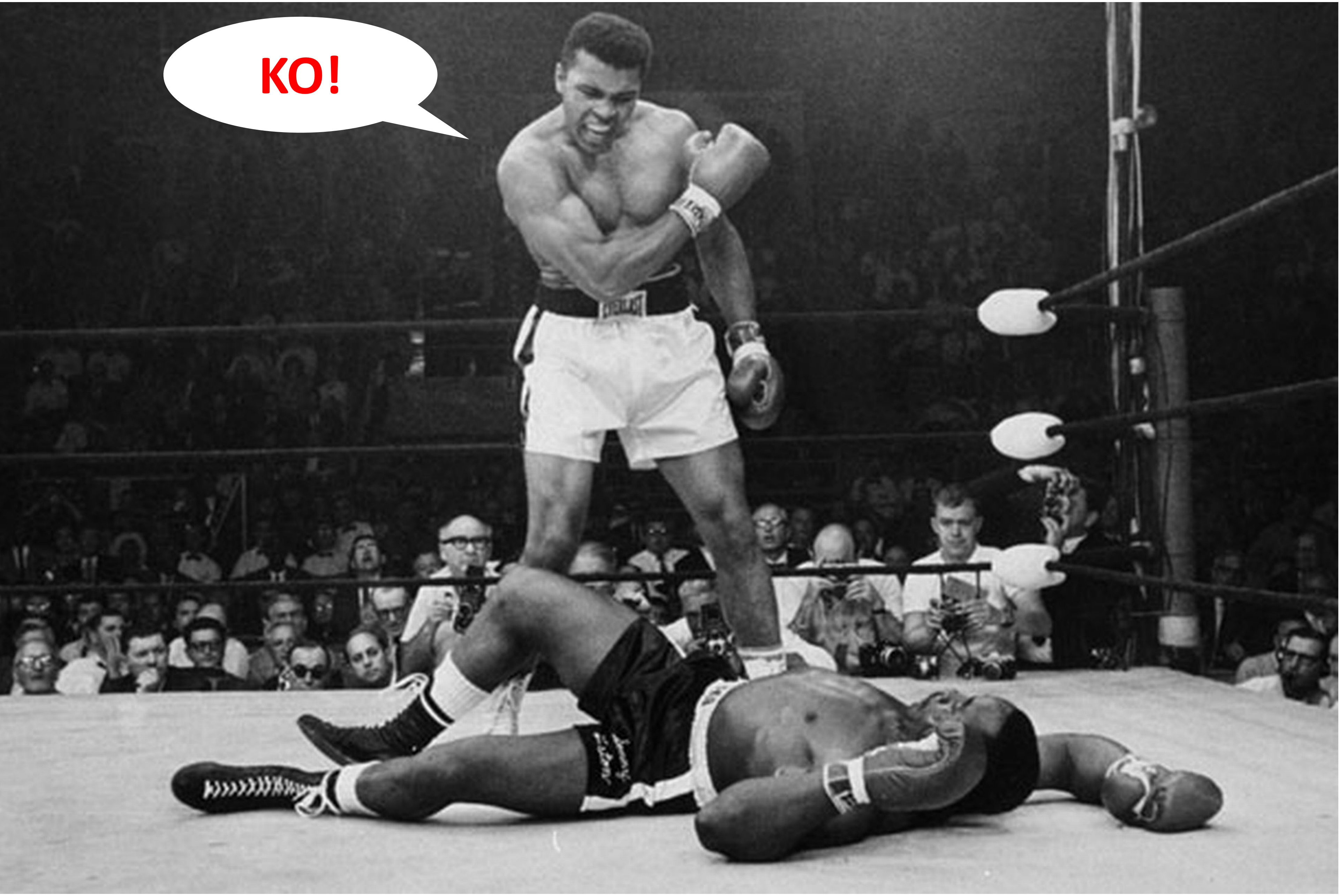
The sky coverage of Ali is double of that of Greenland.



sky coverage: 30%

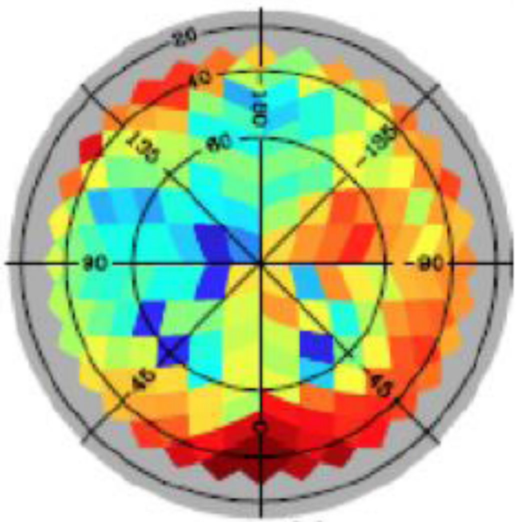


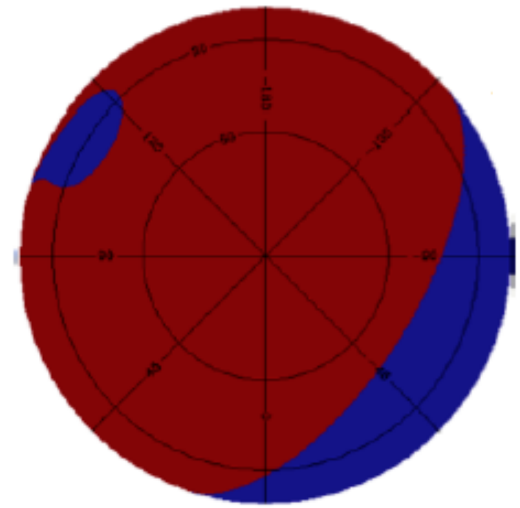
sky coverage: 65%


KO!

Planck results indicate clean areas in the north earth

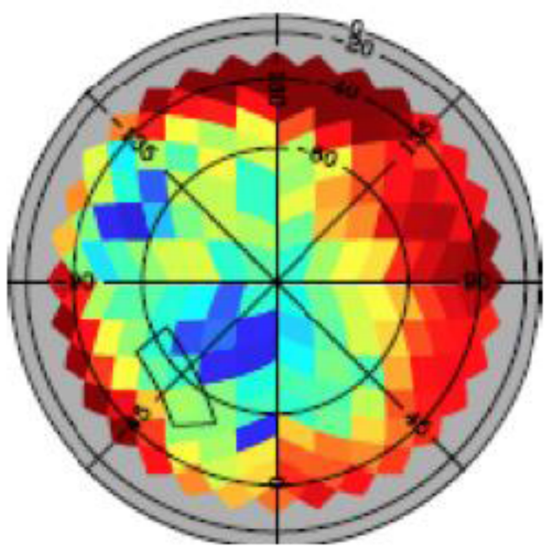
North sphere

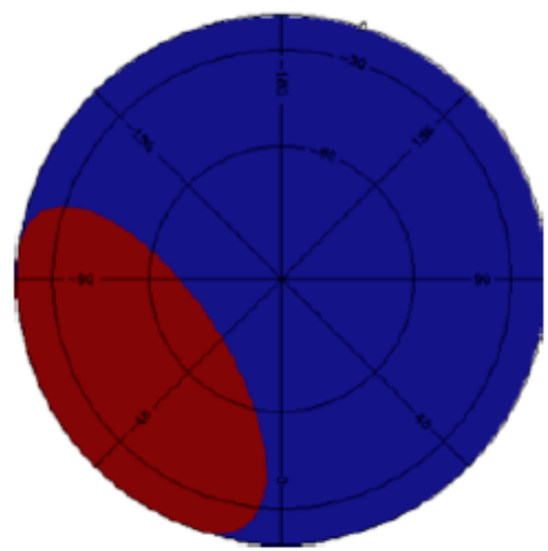




Sky at Ali

South sphere

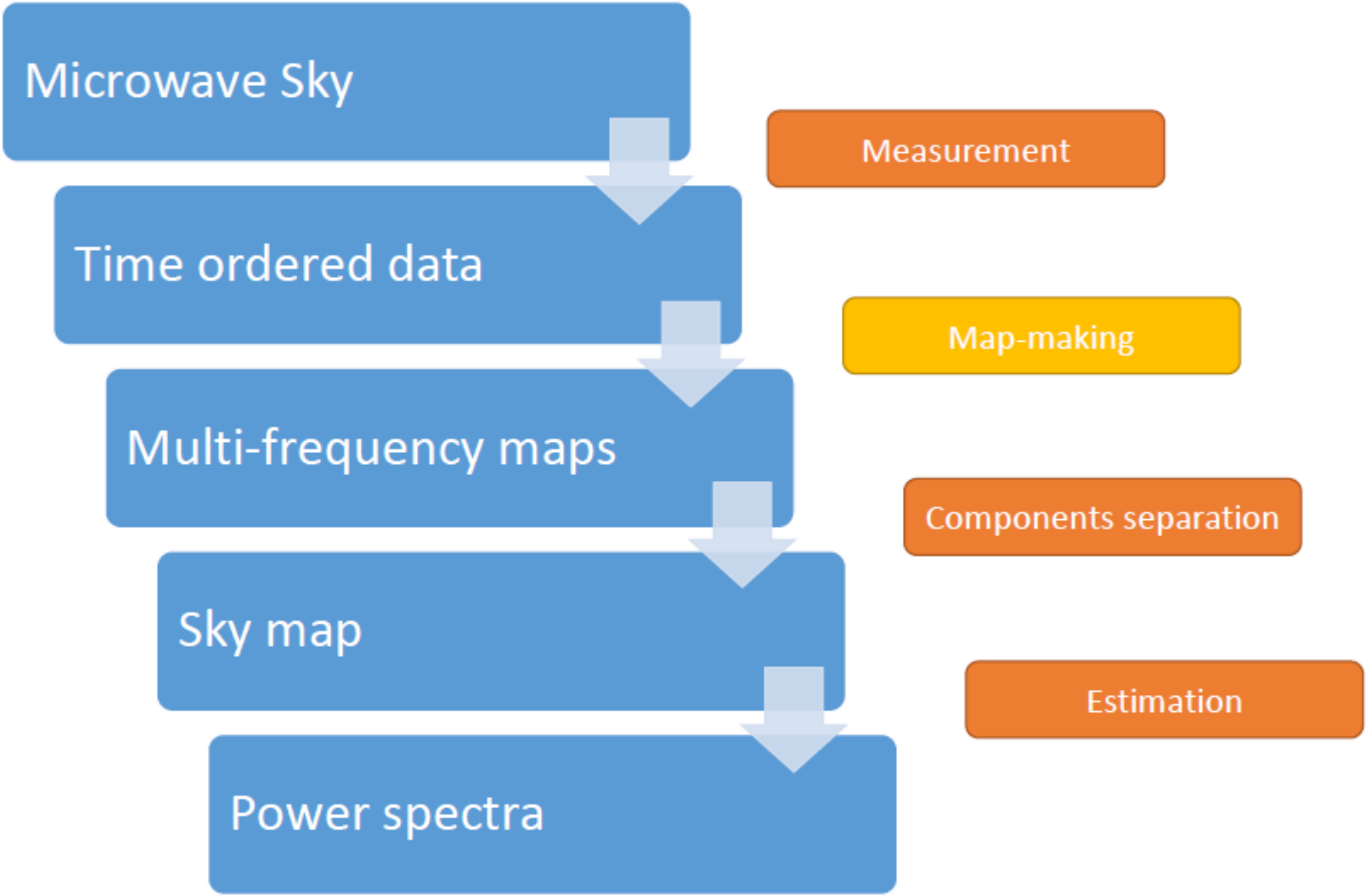




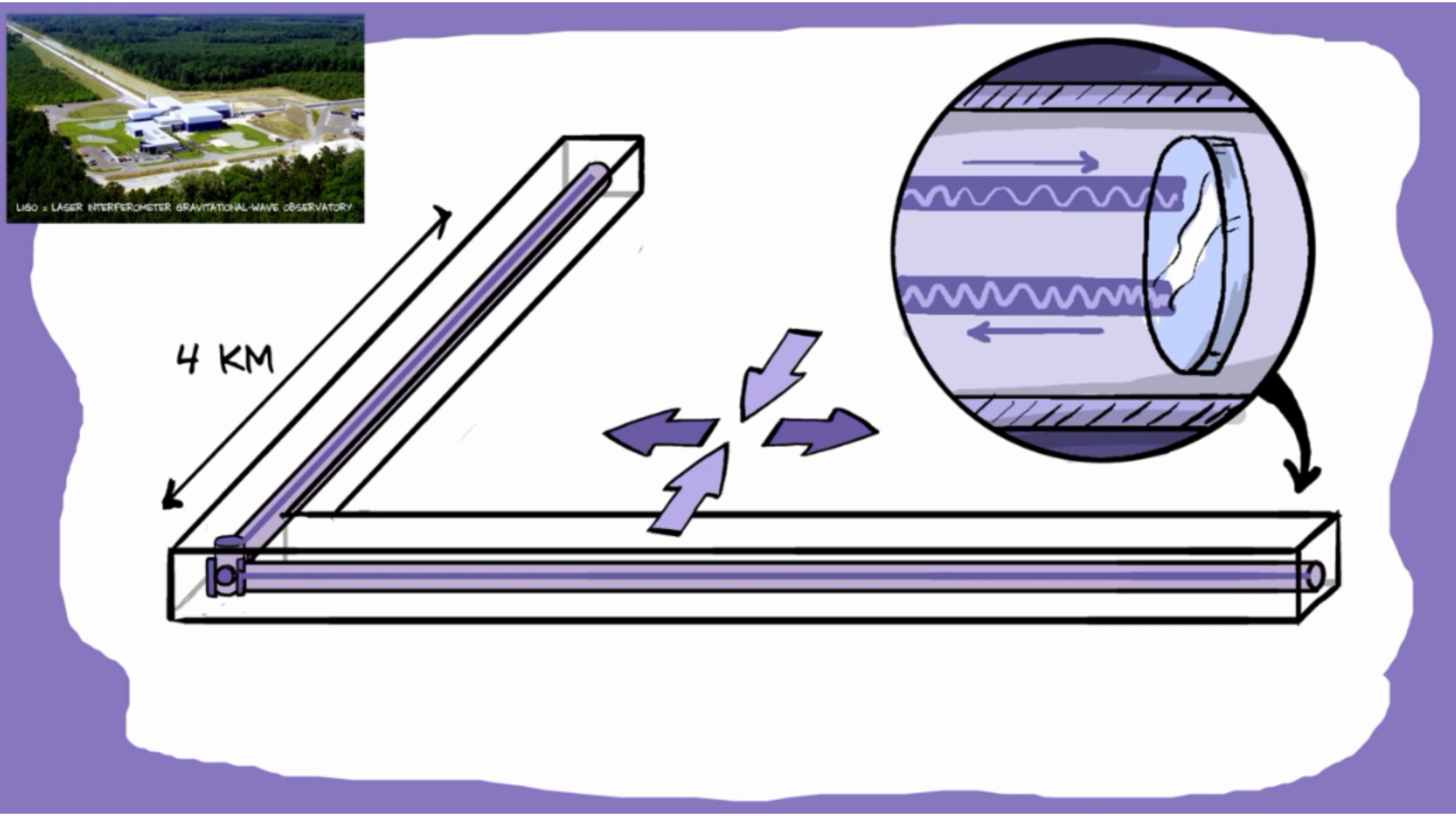
Sky at South Pole


32°19′N 80°1′E
Ali Observatory

Numerical simulations of CMB forecast for Ali Cosmic Polarization Telescopes


Microwave Sky
Time ordered data
Multi-frequency maps
Sky map
Power spectra
Measurement
Map-making
Components separation
Estimation

Comparison between LIGO & Ali



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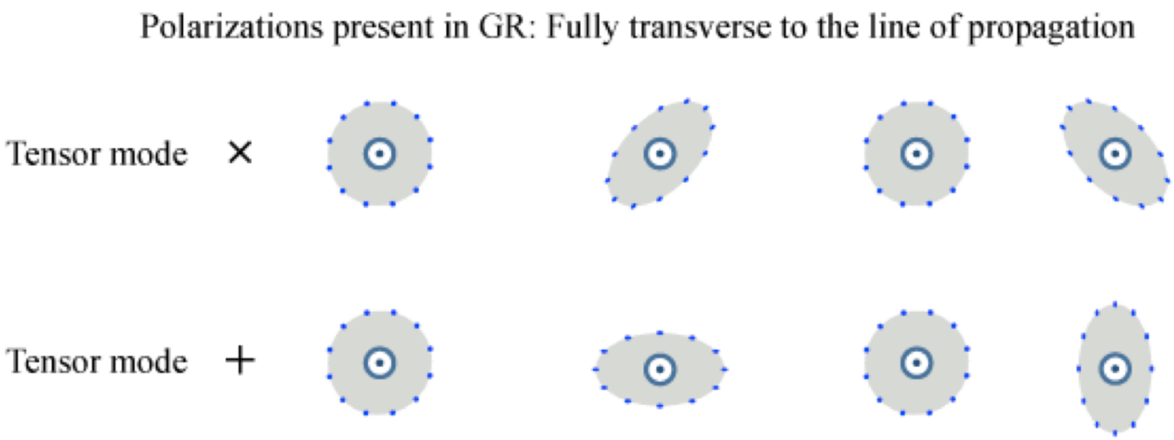
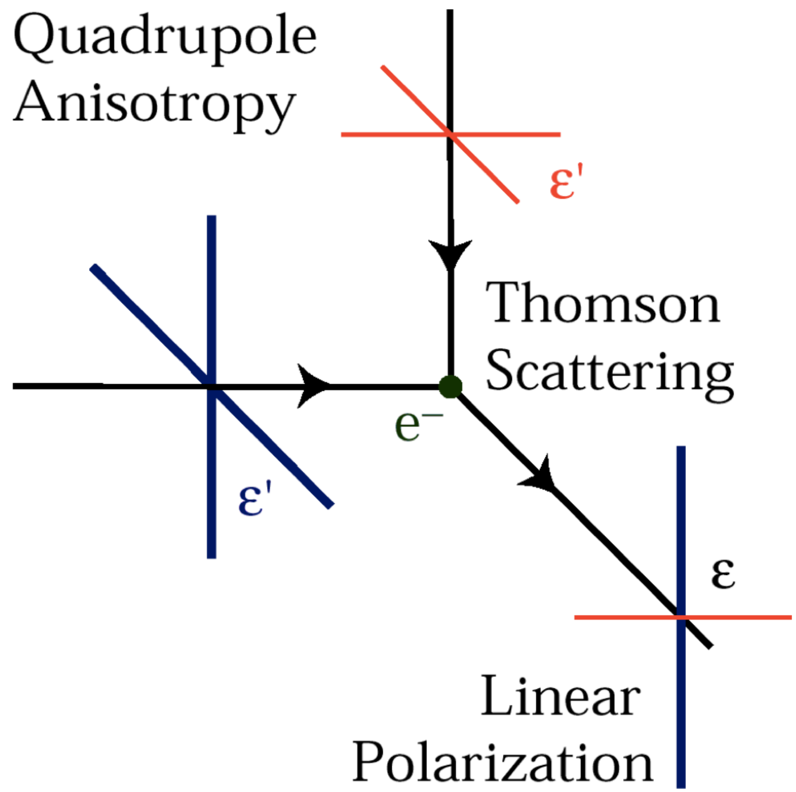
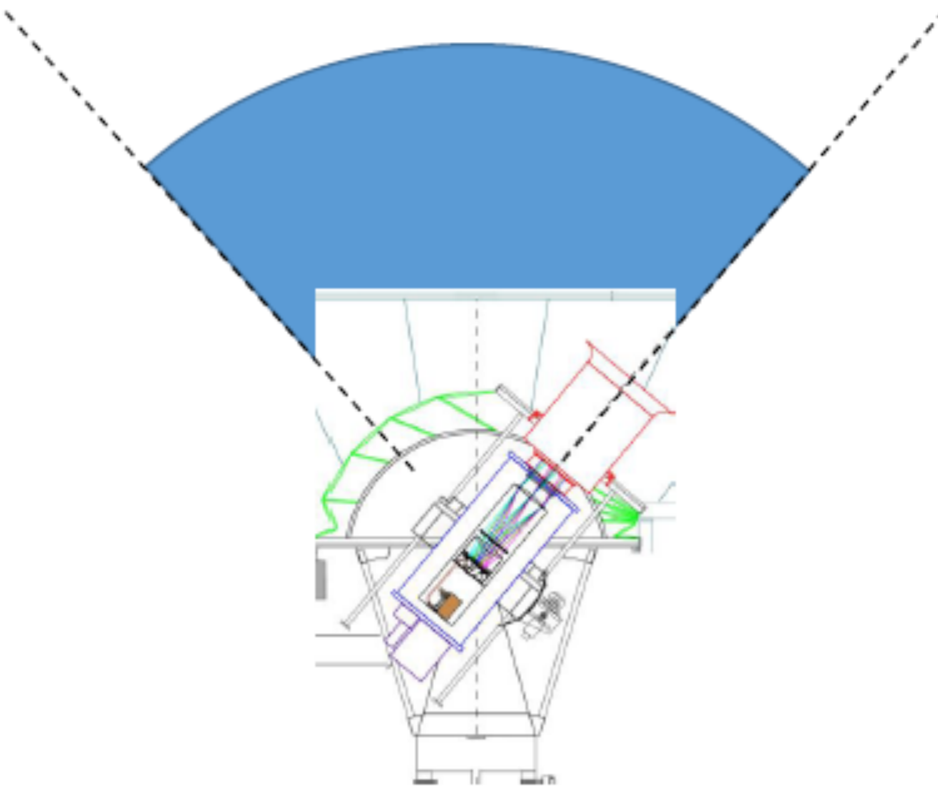




Scan strategy

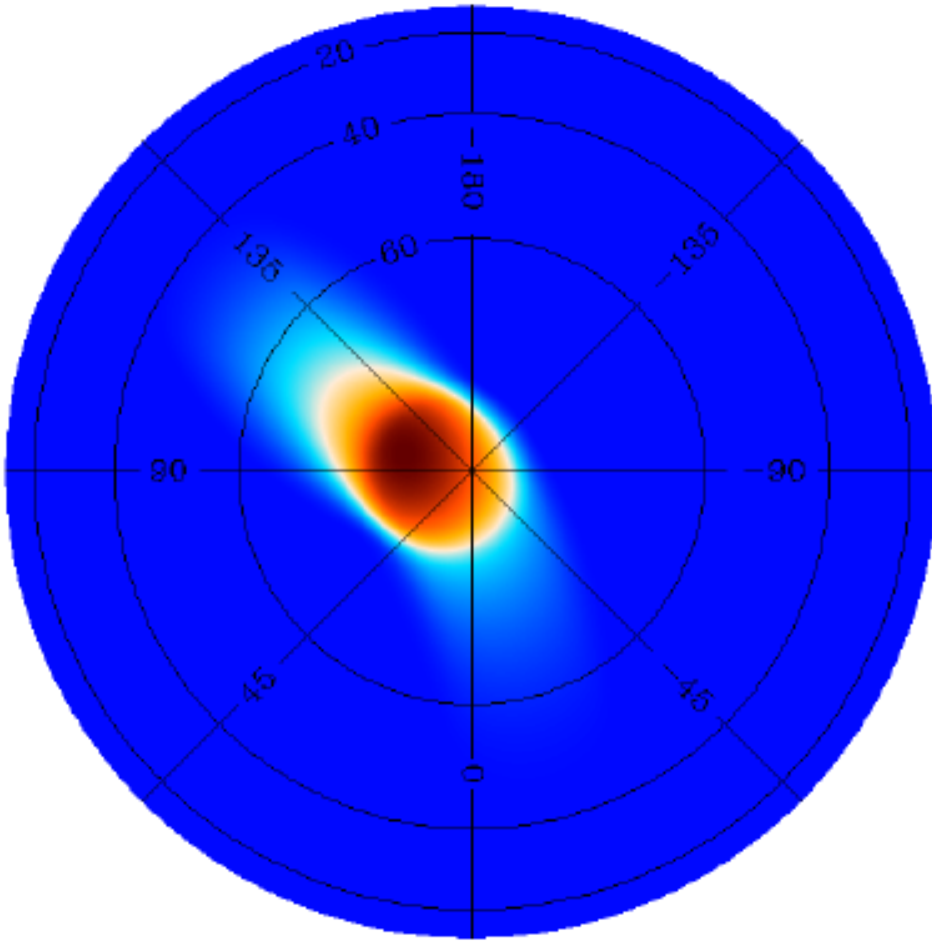
Elevation angle: 45~90 deg



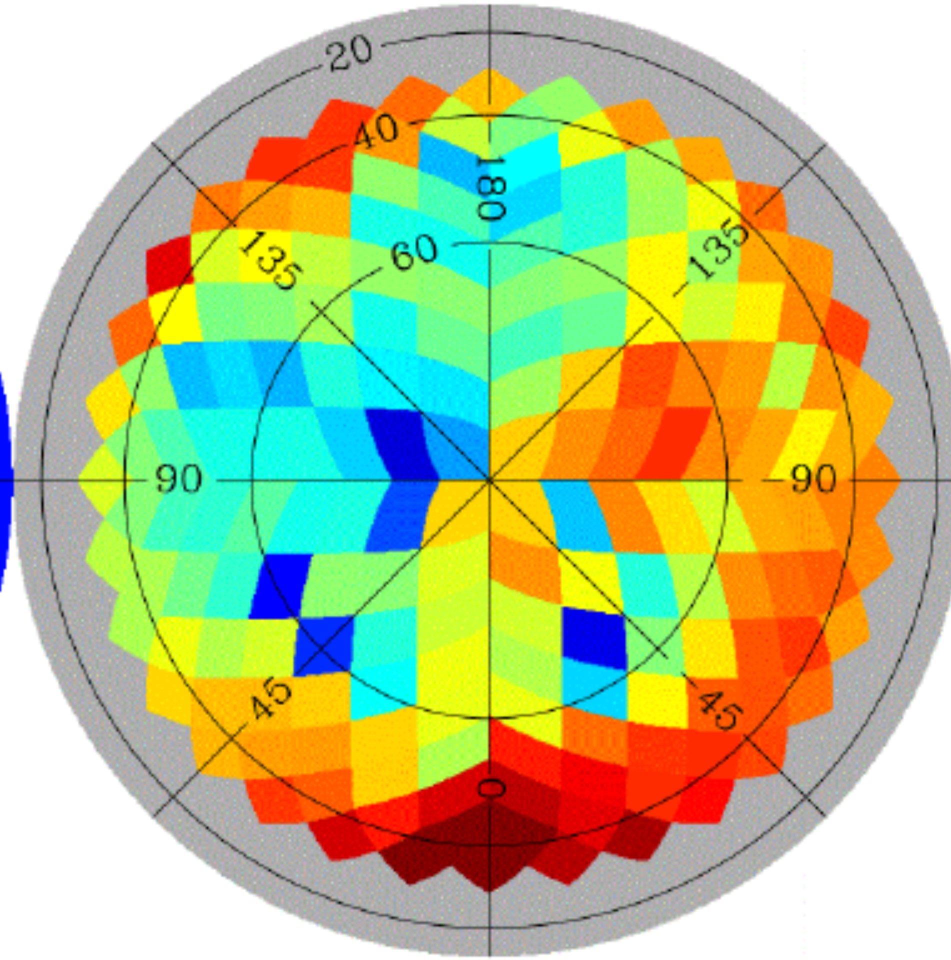
From Nov 25th 2017 to Mar 21st 2018

Observation time: 11 p.m. ~ 5 a.m.

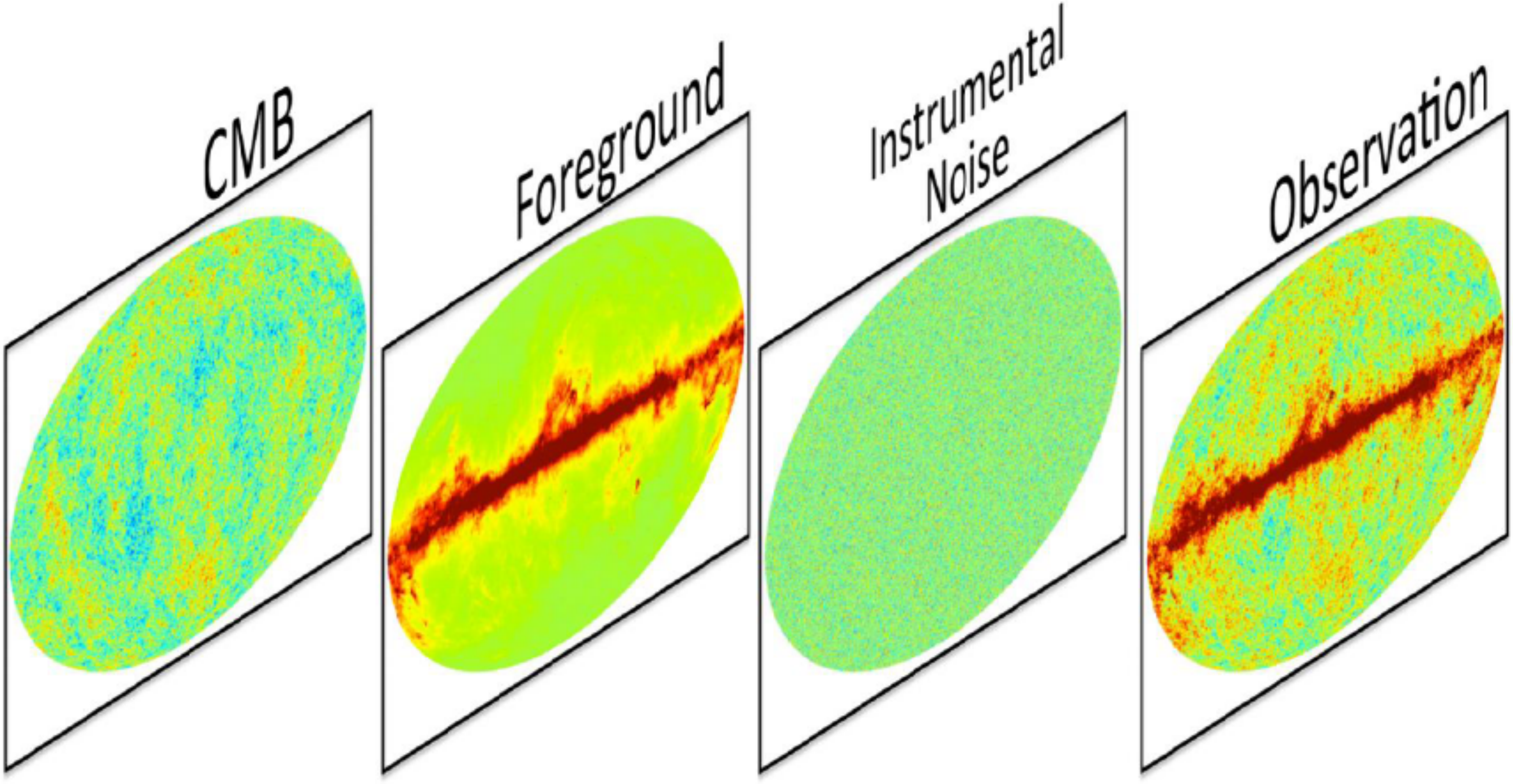
Field of view: 20 deg



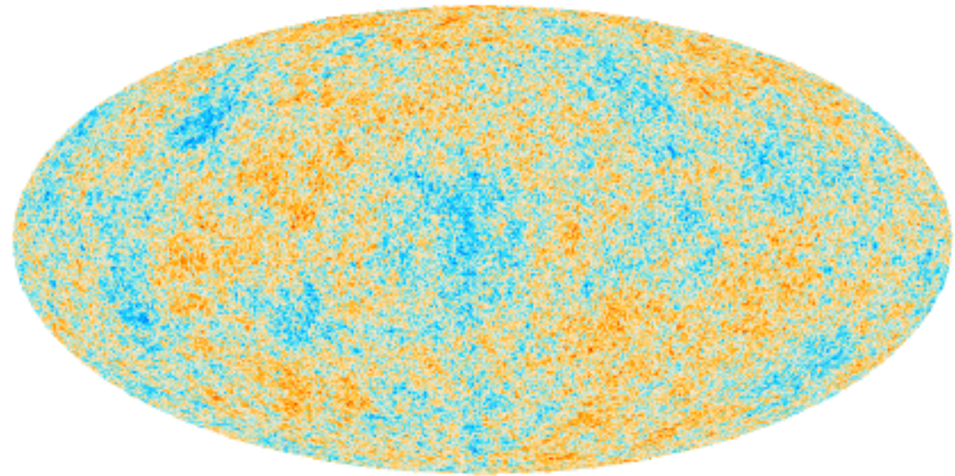
Simulated hit-map

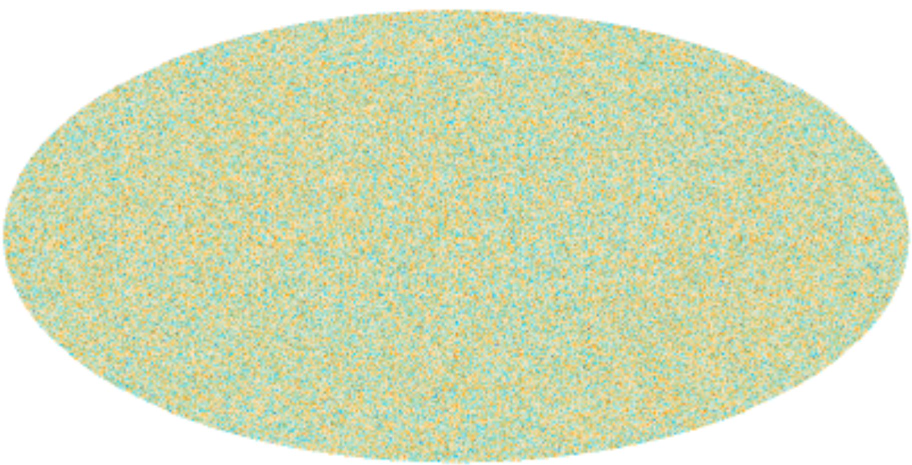


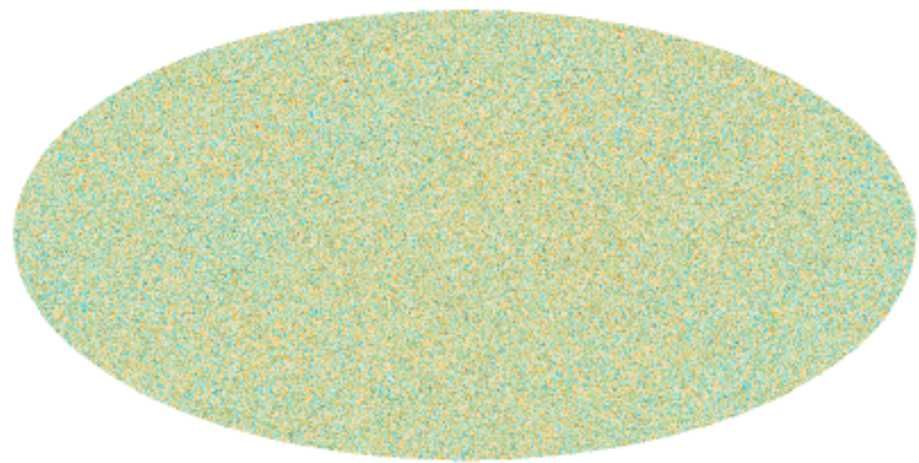
r_{dust} of Planck 353GHz map


CMB
Foreground
Instrumental
Noise
Observation

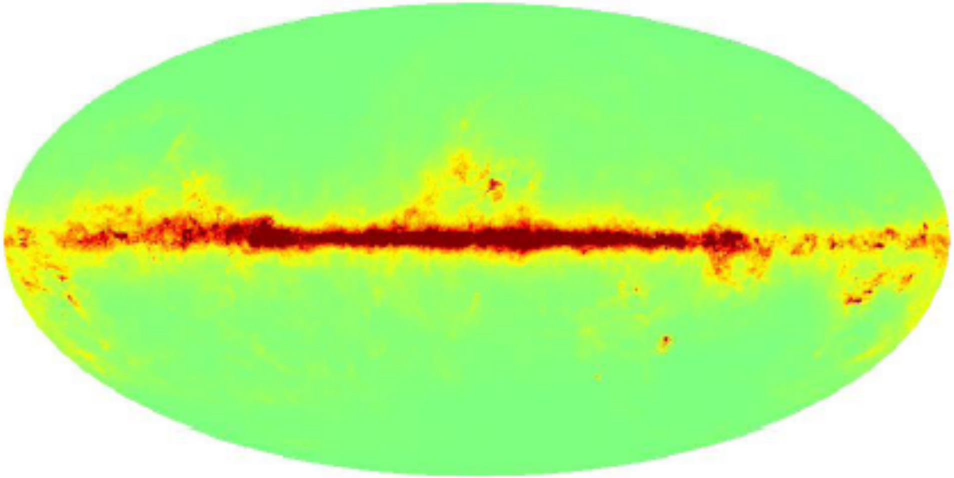
CMB signal maps T Q U

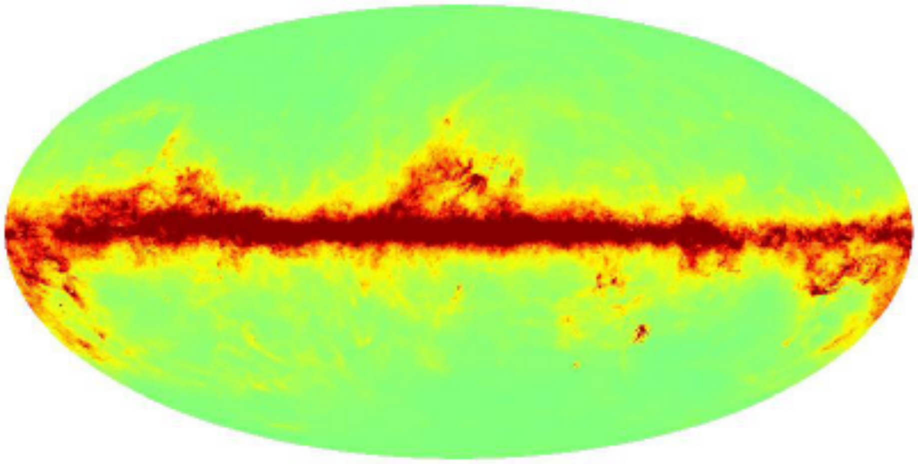


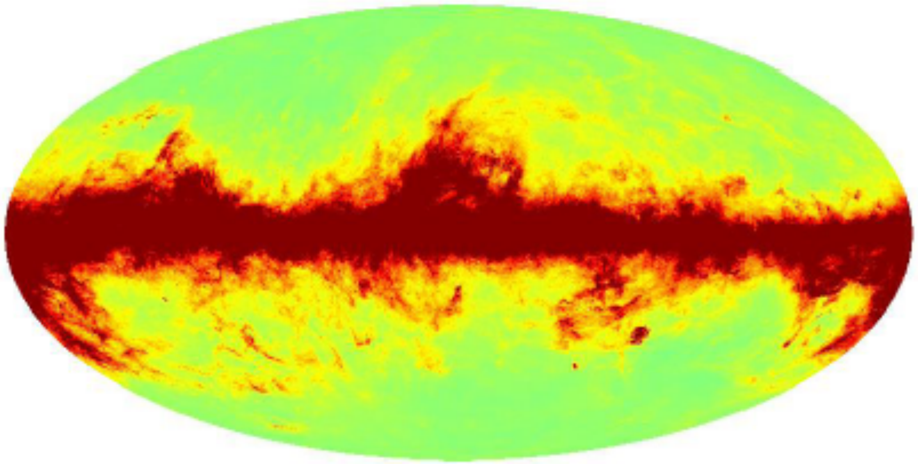




Foreground contamination maps (T)
90Ghz, 150Ghz, 220Ghz







A future lesson from AliCPT

CMB observations of AliCPT @ Tibet

Scientific goals:

1, further depressing the parameter space of very early universe models through the CMB T- & E-mode;

2, attempting to detect primordial gravitational waves through the CMB B-mode;

3, falsifying very early universe models, so that shedding light on the exploration of fundamental physics.

Overview: predictions of very early universe models

Models	Predictions			
Inflation	$r = 16\,\varepsilon$	$n_t = -2\,\varepsilon$	$\alpha_t < 0$	$f_{nl}=5(1-n_s)/12$
Bounce	$r \leq O(1)$	$\lvert n_t \rvert < O(1)$	$\alpha_t > 0$	$f_{nl} \approx -5/2$
Ekpyrosis	$r \ll O(1)$	$n_t = 2$	$\alpha_t < 0$	$f_{nl} > 1$
String gas	$r \leq O(1)$	$n_t \approx 1-n_s$	$\alpha_t > 0$	$f_{nl} \ll O(1)$

Any bonus from AliCPT?

- To measure the spectral and running indices of primordial gravitational waves;

- To test CMB anomalies noticed by WMAP & Planck:
  - low ell suppression
  - CMB power asymmetry
  - l=20 power deficit
  - cold spot, …

- To improve the constraint of the nonlinearity parameter;

- To measure the E-mode in the north sky;

- …

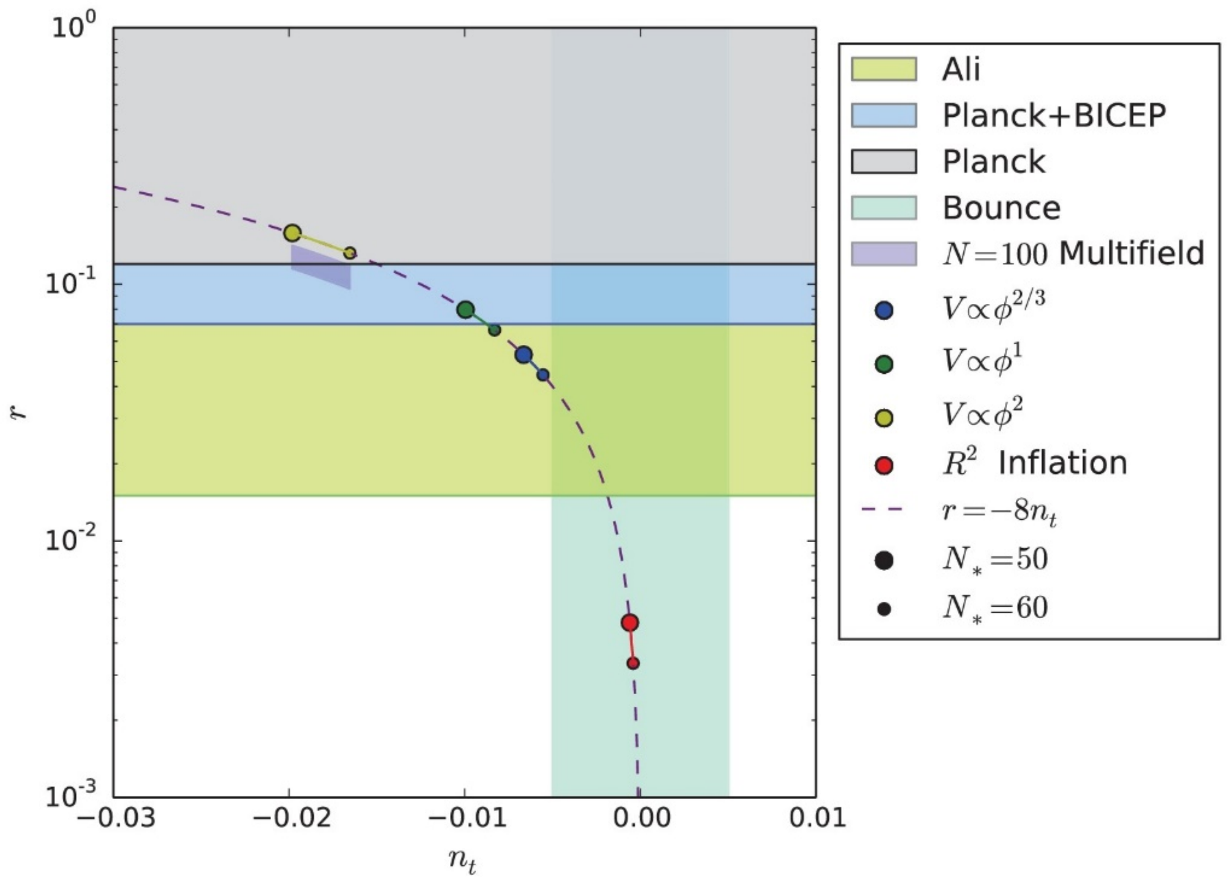
AliCPT at USTC

- Theoretical analyses of primordial gravitational waves
  - Constraining the very early universe models
  - Statistics of all possible B-mode components
  - CMB large scale anomalies
- CMB B mode testing CPT symmetry
  - Using CMB data to test CPT
  - De-rotation analysis of CMB polarizations

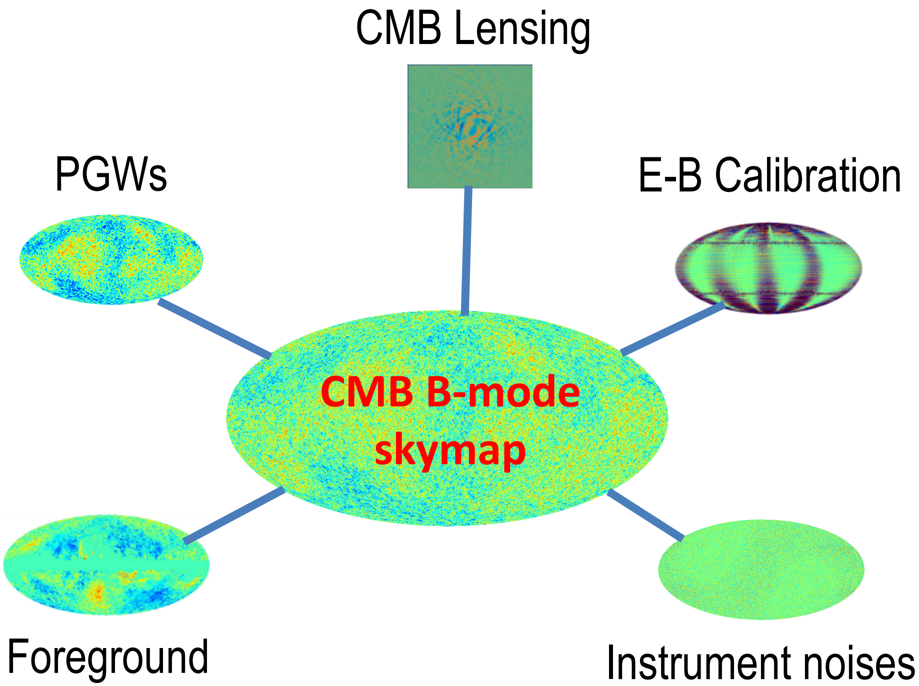
Constraining the very early universe models

Q: there are too many models of the very early universe, namely,

- Inflationary models
- Nonsingular bounce models
- Models of emergent universe



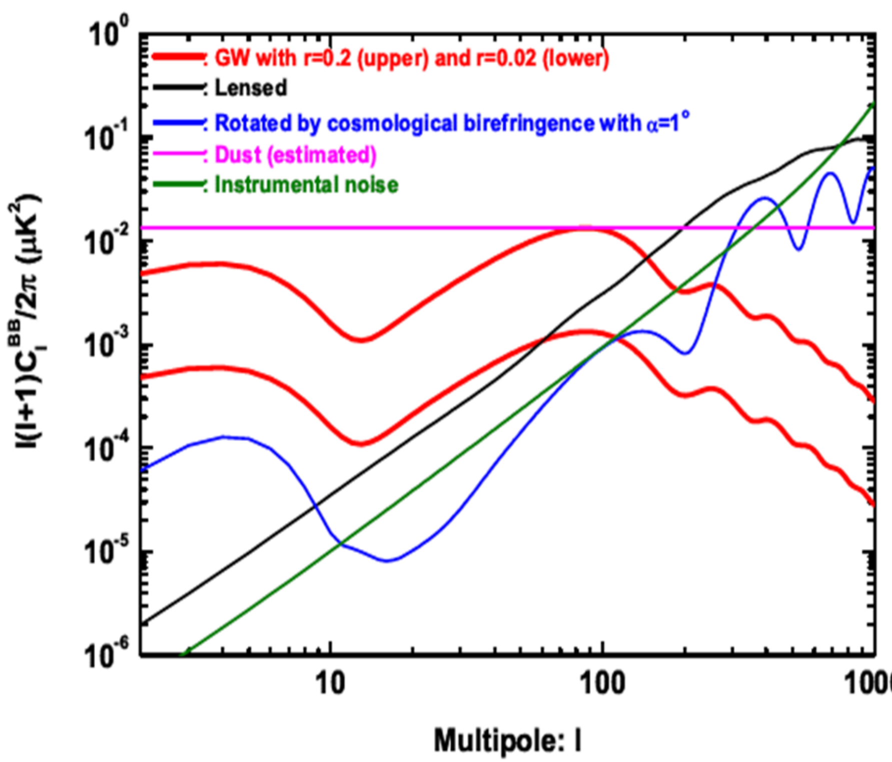
Statistics of all possible B-mode components



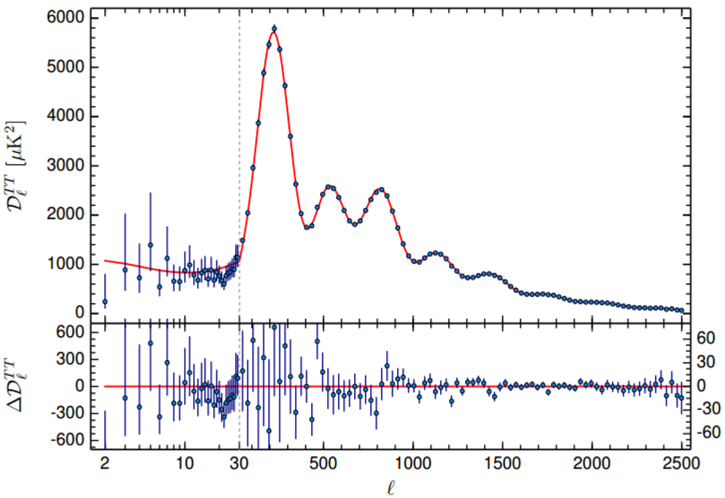
Q: How can we identify all the components that can give rise to CMB B-mode?

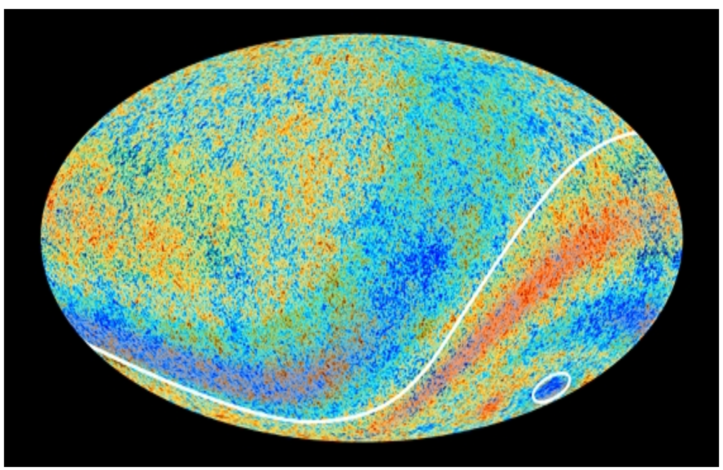
Plan：

Different components exhibit different statistical properties. These can be used to exact the signals from PGWs.



CMB large scale anomalies





- Large scale suppression
- Cold spot
- Hemispherical power asymmetry
- Power deficit near l=30

Q: primordial origin, or, observational contamination?

Plan:

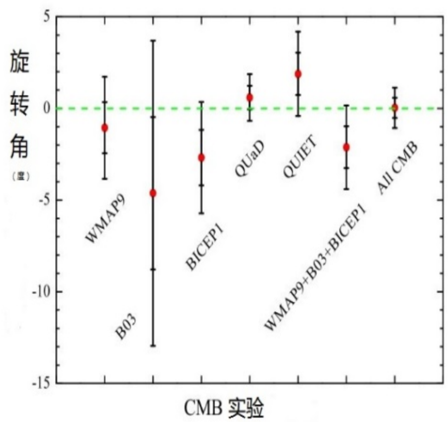
✓ Combine together AliCPT in North sphere and BICEP, PolarBear in South sphere

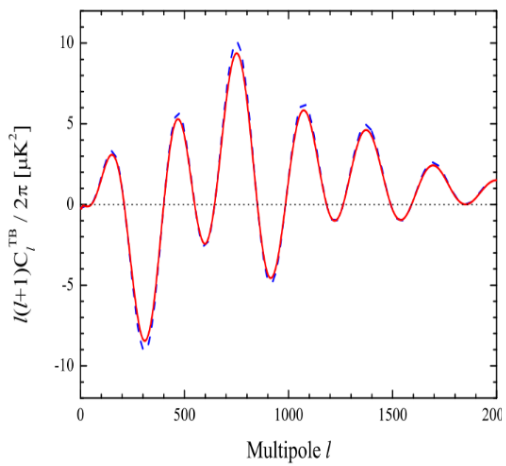
✓ Build theoretical models to explain associated phenomena

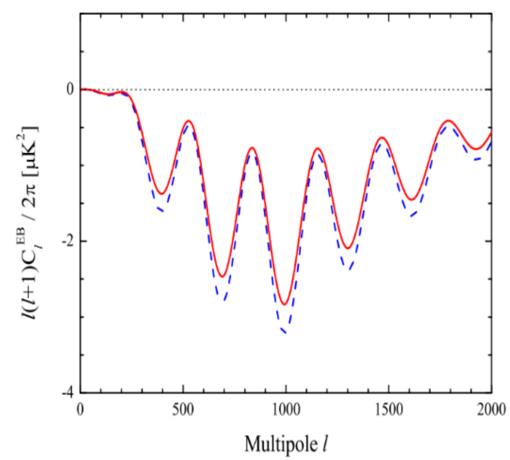
Using CMB data to test CPT

Q: Charge-Parity-Time reverse symmetry is viewed as the fundamental symmetry in particle physics. Is it true throughout the whole cosmological history? If not, how can we probe the associated violation?

➔Rotation between E and B modes of CMB







Plan:

- Develop the theory of CPT violation
- Mimic the observational signals in AliCPT
- Establish the effective field theory
- Parameterize the rotation angle using cosmological parameters

De-rotation analysis of CMB polarizations

CPT violation can lead to the rotation between E and B modes, and thus, this part of contribution to CMB B-mode can be categorized as foreground contamination.

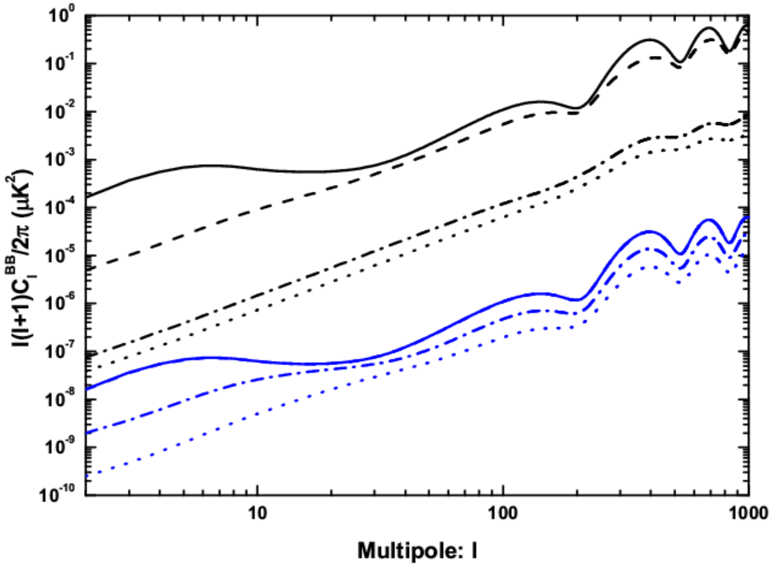
Q: How can we de-rotate the E and B modes that were induced by CPT violation?

Plan: In analogue with the CMB lensing technique, we can reconstruct the rotation angle by virtue of the data of T and E modes. Then, we can perform the de-rotation analysis with the AliCPT data in future.

$$\hat{B}_{\ell m} = \sum_{\ell_2 m_2} \sum_{\ell_3 m_3} f(\ell, m, \ell_2, m_2, \ell_3, m_3) \hat{\alpha}_{\ell_2 m_2} \hat{E}_{\ell_3 m_3}$$

$$\Downarrow$$

$$C_\ell^{BB}(\text{residual}) = \frac{1}{\pi} \sum_{\ell_2 \ell_3} (2\ell_2+1)(2\ell_3+1) C_{\ell_2}^{\alpha\alpha} C_{\ell_3}^{EE} (H_{\ell \ell_3}^{\ell_2})^2 \left(1 - \Theta_{\ell_2 \ell_3}\right)$$



Summary & Outlook

Today

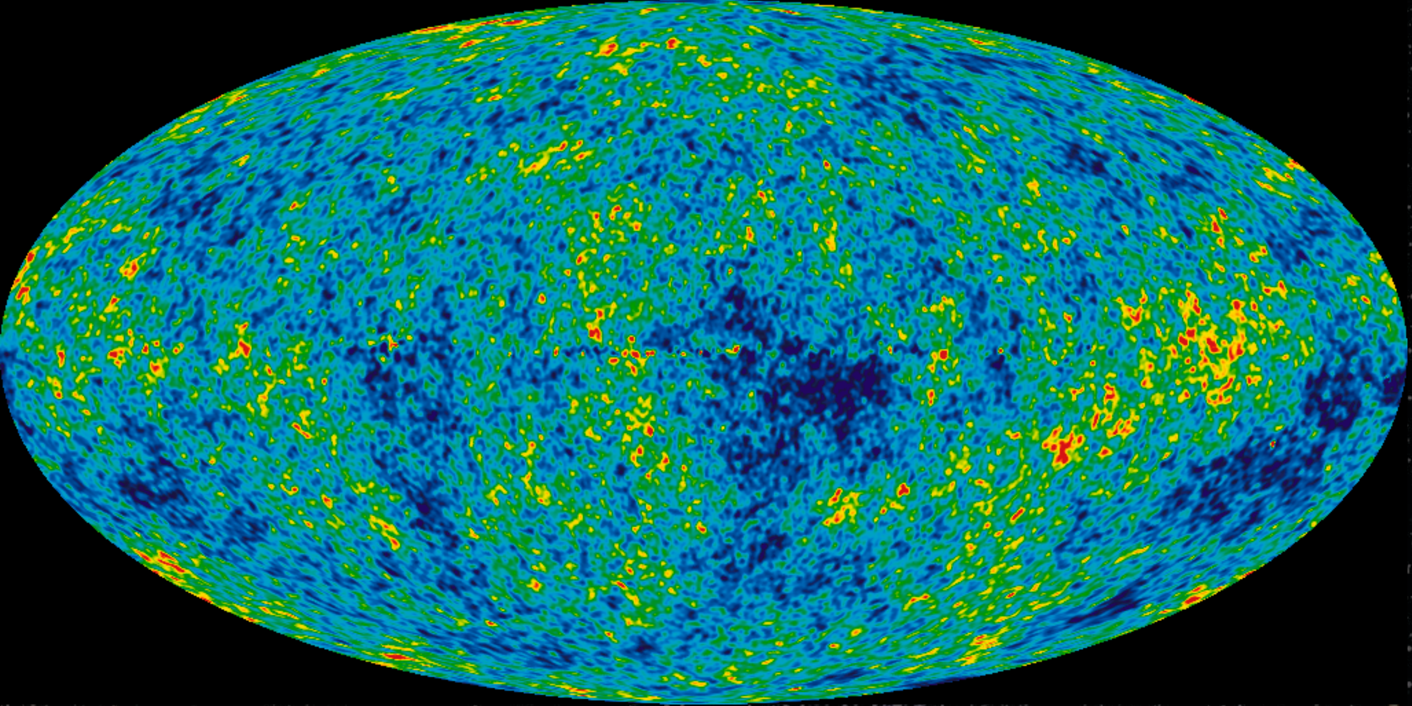
- The past decade has witnessed the era of precision cosmology
- The paradigm of early universe has been greatly developed
- Big Bang cosmology has become the Standard Model
- Inflation obtained a large amount of initial achievements
- Bounce cosmology is ambitious on solving big bang singularity

In Near Future

- Very early universe opens a window to explore fundamental physics
- It becomes possible to observationally probe physics near the Big Bang: CMB experiments
- CMB experiments in the north earth is necessary
- China's Ali CMB project will be the first CMB experiment in the north earth

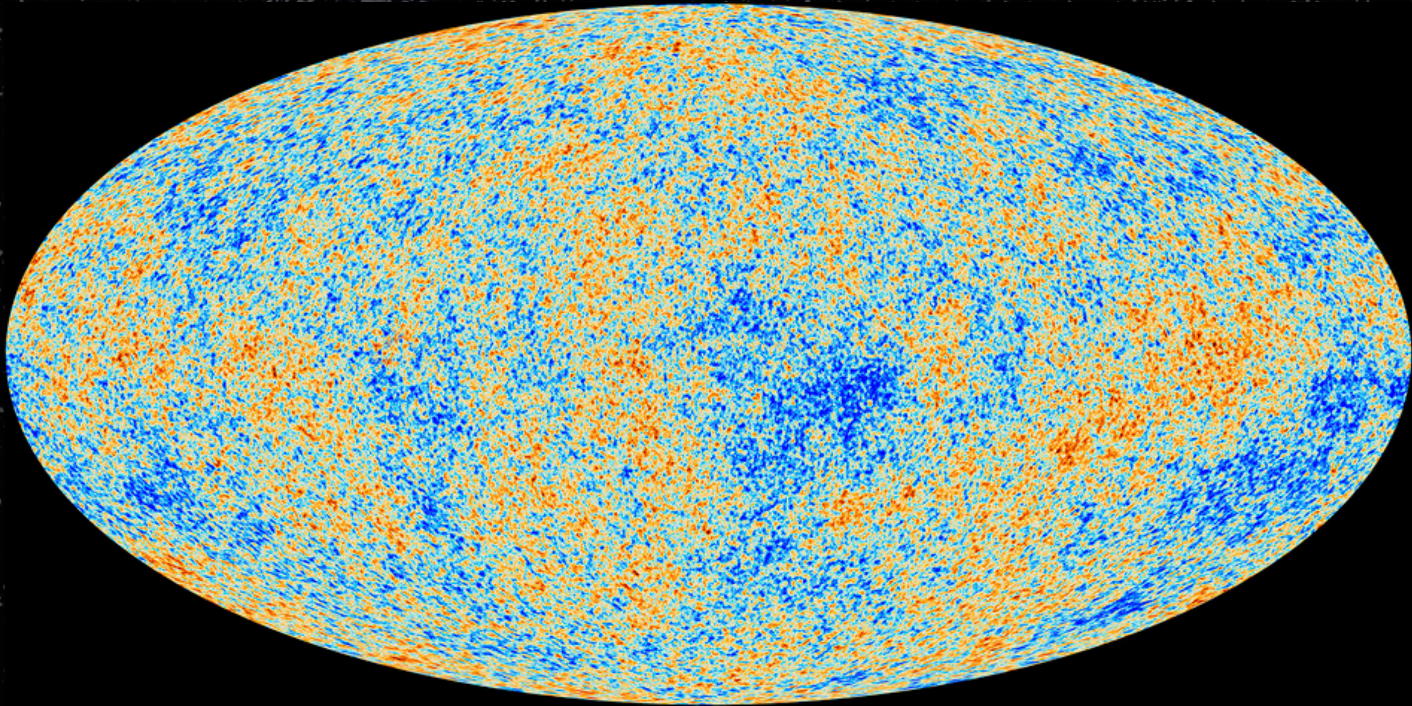
This is just the beginning …

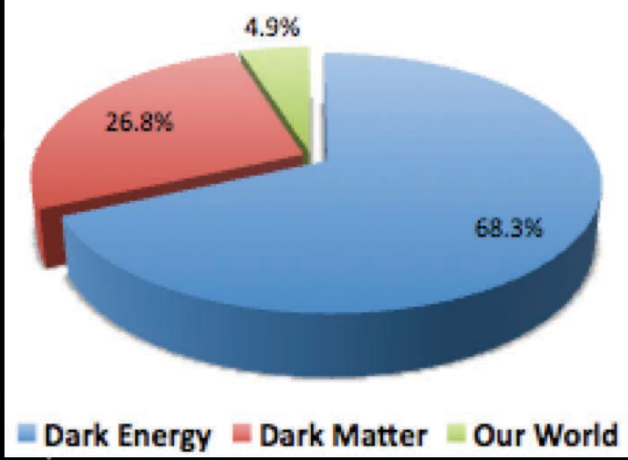
Thanks!



WMAP/NASA

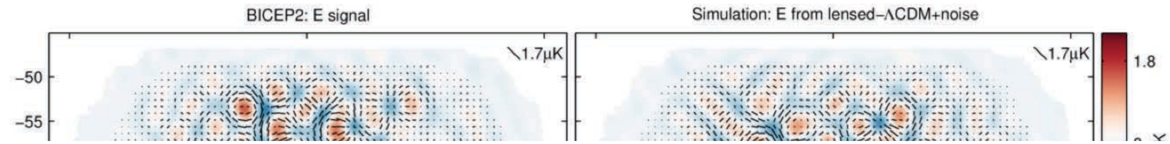
Planck/ESA

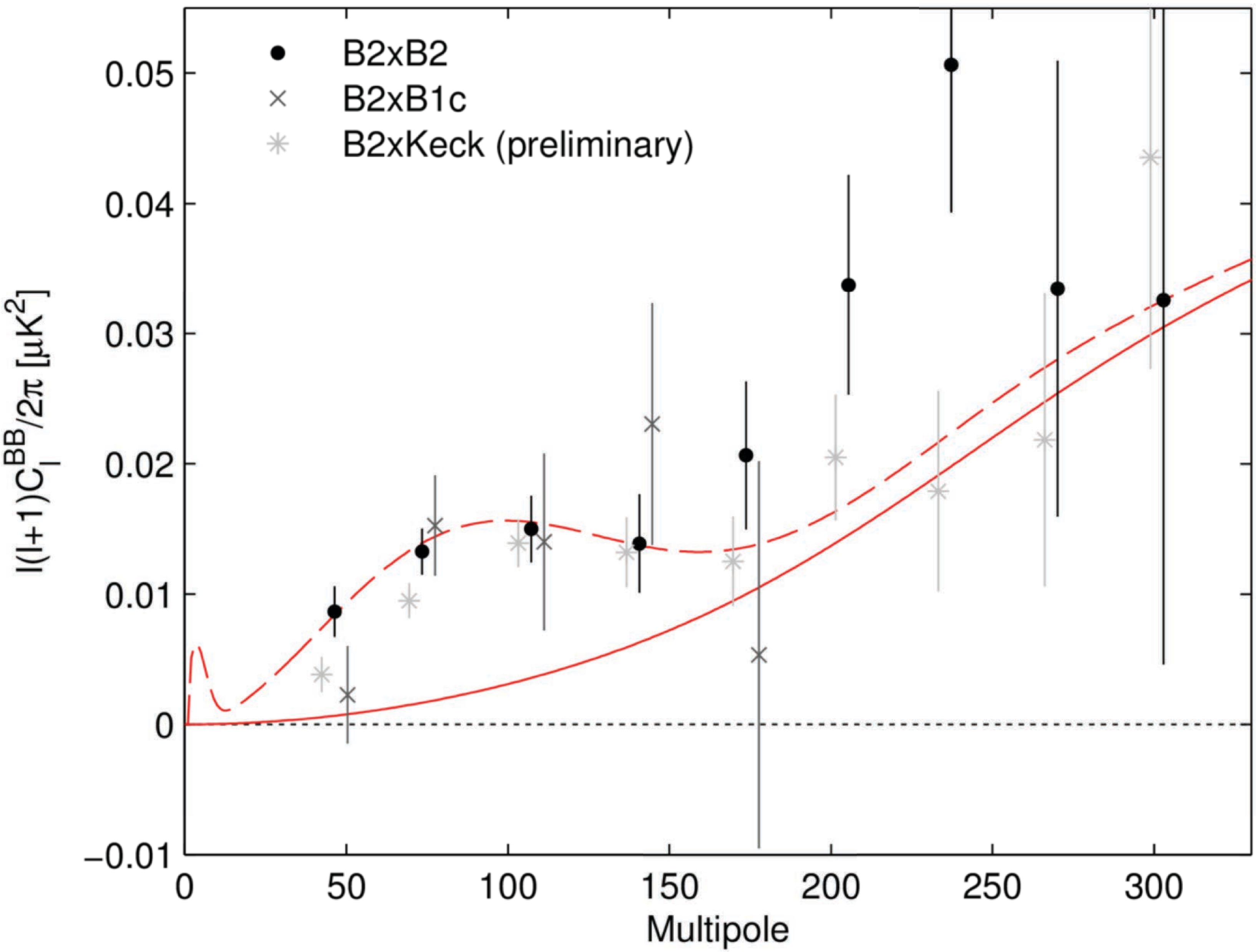


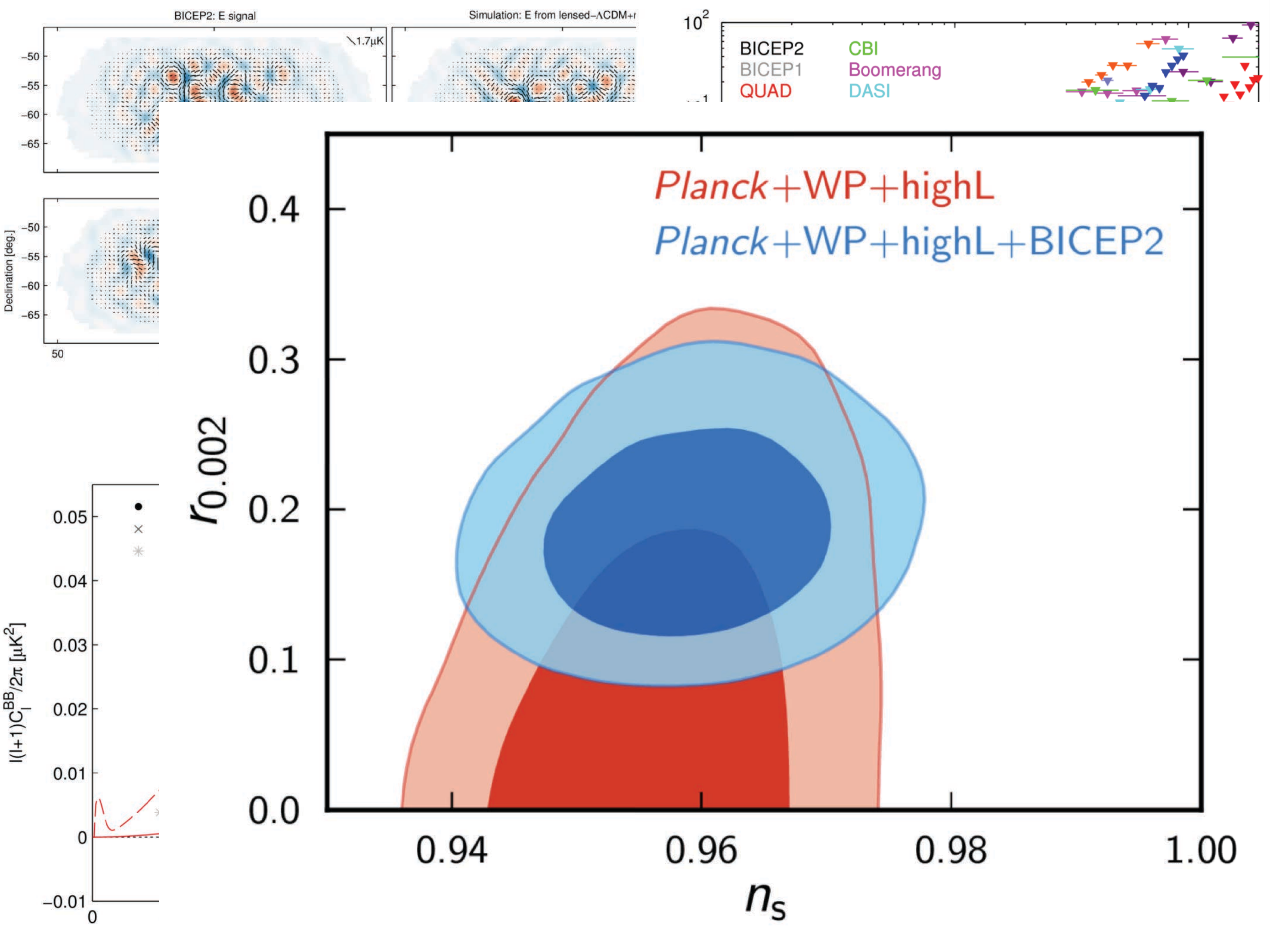


Observational Facts:

- Our universe has a thermal expanding history with 13.8 billion years
- The background looks the same at anywhere on sufficiently large scales
- Galaxies and clusters are basic blocks to form the LSS

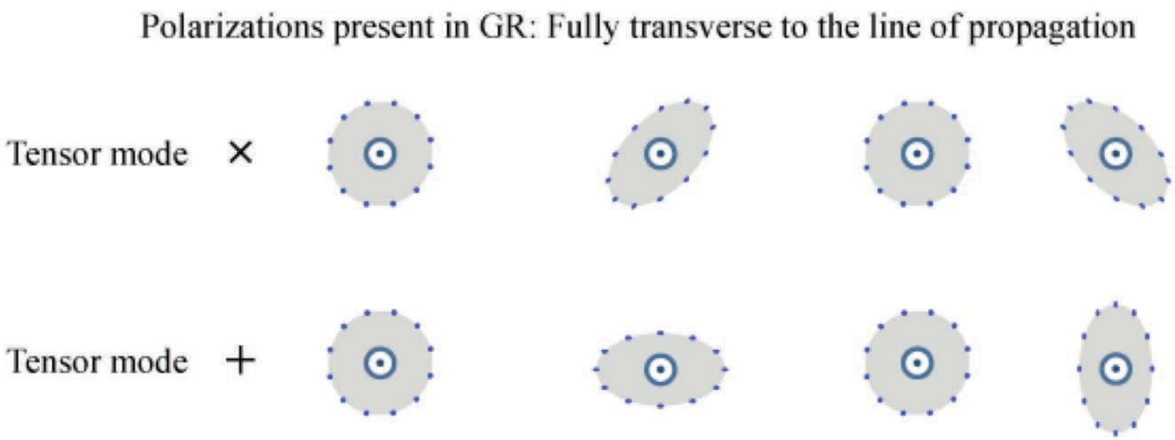

BICEP2: E signal
Simulation: E from lensed-ΛCDM+noise
1.7μK
1.7μK
1.8
−50
−55

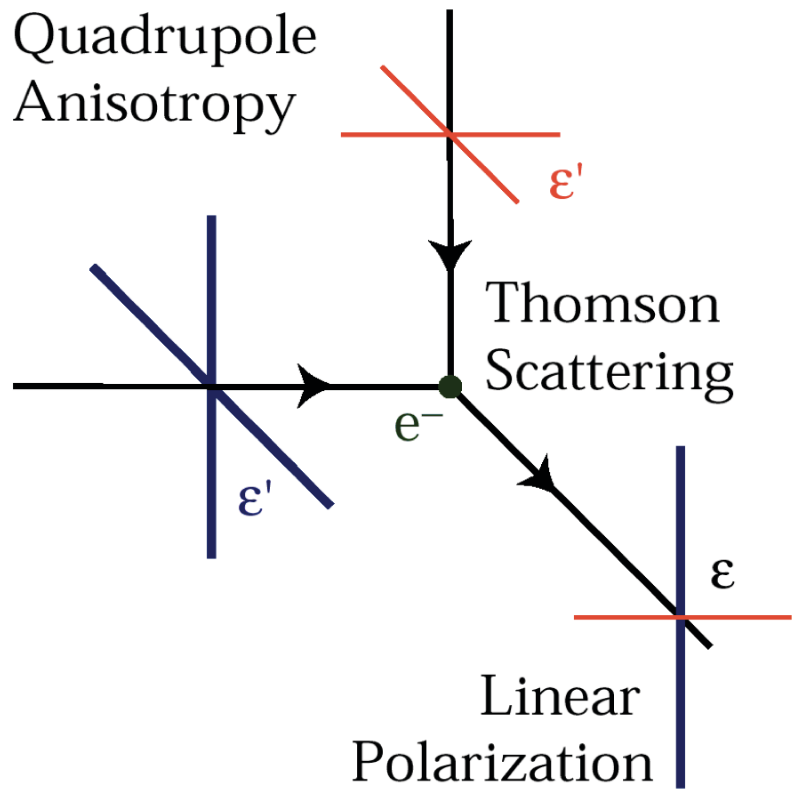

B2xB2
B2xB1c
B2xKeck (preliminary)
l(l+1)C_l^BB/2π [μK²]
Multipole
0.05
0.04
0.03
0.02
0.01
0
−0.01
0
50
100
150
200
250
300



BICEP2: E signal
Simulation: E from lensed-ΛCDM+r
1.7μK
Declination [deg.]
BICEP2
BICEP1
QUAD
CBI
Boomerang
DASI
Planck+WP+highL
Planck+WP+highL+BICEP2
$r_{0.002}$
n_s
0.94
0.96
0.98
1.00
0.0
0.1
0.2
0.3
0.4
$l(l+1)C_l^{BB}/2\pi$ [μK²]

Comparison between LIGO & Ali





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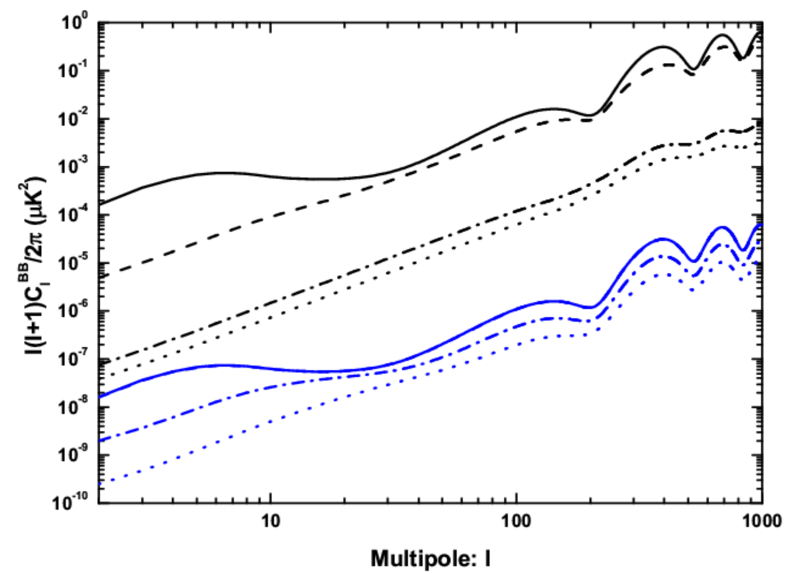

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