A unified model for galactic discs: star formation, turbulence driving, and mass transport

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Energy budget

The turbulent energy per unit area contained in a galactic disc of gas surface density Σ_g and velocity dispersion σ_g is

$$\left(\frac{\mathrm{d}E}{\mathrm{d}A}\right)_{\mathrm{turb}} \approx \frac{3}{2}\Sigma_g\sigma_g^2 = 3.1\times10^9\,\Sigma_{g,10}\sigma_{g,10}^2\ \mathrm{erg\ cm^{-2}}, \tag{1}$$

where $\Sigma_{g,10} = \Sigma_g/10\,\mathrm{M_\odot\ pc^{-2}}$ and $\sigma_{g,10} = \sigma/10\,\mathrm{km\,s^{-1}}$; the scaling factors are typical values at the Solar Circle in the Milky Way. The energy should dissipate due to decay of turbulence over a time-scale comparable to the crossing time, but, in a disc with $Q \approx 1$, this time-scale is comparable to the galactic dynamical time $t_{\mathrm{dyn}} = r/v_\varphi$,

$$\frac{\mathrm{d}^2 E}{\mathrm{d}t\,\mathrm{d}A} \approx \frac{\dot{M}_{\mathrm{in}} v_\phi^2}{2\pi r^2}, \tag{2}$$

so over a galactic dynamical time the flow delivers an energy per unit area

$$\left(\frac{\mathrm{d}E}{\mathrm{d}A}\right)_{\mathrm{inflow}} \approx \frac{\dot{M}_{\mathrm{in}} v_\phi}{2\pi r} = 6.5 \times 10^9\, \dot{M}_{\mathrm{in},1} v_{\phi,200} r_{10}^{-1}\ \mathrm{erg\ cm^{-2}}, \tag{3}$$

$$\begin{aligned}\left(\frac{\mathrm{d}E}{\mathrm{d}A}\right)_{\mathrm{sf}} &\approx \dot{\Sigma}_* \left\langle \frac{p_*}{m_*} \right\rangle \sigma_{\mathrm{g}} \frac{r}{v_\phi} \\ &= 3.1 \times 10^9\, \dot{\Sigma}_{*,-3} \sigma_{\mathrm{g},10} r_{10} v_{\phi,200}^{-1}\ \mathrm{erg\ cm^{-2}},\end{aligned} \tag{5}$$

where $\dot{\Sigma}_{*,-3} = \dot{\Sigma}_*/10^{-3}\ \mathrm{M_\odot\ pc^{-2}\ Myr^{-1}}$.

quite different results. The ratio of $(\mathrm{d}E/\mathrm{d}A)_{\mathrm{inflow}}$ to $(\mathrm{d}E/\mathrm{d}A)_{\mathrm{sf}}$ scales as $(\dot{M}_{\mathrm{in}}/\dot{M}_*)(v_\phi^2/\sigma_{\mathrm{g}})$, where $\dot{M}_*$ is the total star formation rate. We do not have direct measurements of $\dot{M}_{\mathrm{in}}$ except in local spirals, but assuming that $\dot{M}_{\mathrm{in}}/\dot{M}_* \sim 1$, as would be required to explain star formation fuelling and as is observed locally, star formation should be energetically dominant in galaxies with smaller v_φ (for example local dwarfs), while inflow should dominate those with larger σ_{g} (for example high-z galaxies). Clearly it is not reasonable to ignore

Gravitational instability

$$Q = f_{g,Q} Q_g,$$

where

$$f_{g,Q} \equiv \frac{\Sigma_g}{\Sigma_g + [2\sigma_g^2/(\sigma_g^2 + \sigma_*^2)]\Sigma_*}.$$

$$Q_g = \frac{\kappa \sigma_g}{\pi G \Sigma_g}$$

In the Solar neighbourhood, fg,Q ≈ 0.6

Due to the fact that gas is dissipational and thus usually has a lower velocity dispersion than stars, it tends to be the most unstable component in any gasrich system. Thus we expect Q∗ > Qg to hold in local dwarfs and lower mass spirals, all star-forming galaxies at high redshift, and in all mergers and starbursts.

Vertical force balance

- 垂向金斯方程

$$\rho_{\mathrm{g,mp}}\left(\sigma_{\mathrm{g}}^2+v_{\mathrm{A}}^2\right)_{\mathrm{mp}}=-\int_0^{\infty}\langle\rho_{\mathrm{g}}g_z\rangle\,\mathrm{d}z,$$

- 泊松方程

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right)+\frac{\partial^2\psi}{\partial z^2}=4\pi G\rho,$$

$$\rho_{\mathrm{g,mp}}\left(\sigma_{\mathrm{g}}^2+v_{\mathrm{A}}^2\right)_{\mathrm{mp}}\equiv\phi_{\mathrm{mp}}\rho_{\mathrm{g,mp}}\sigma_{\mathrm{g}}^2, \tag{12}$$

where φ_{mp} is a factor that represents the factor by which the mid-plane pressure exceeds that due to turbulent plus thermal pressure alone, due to magnetic and cosmic ray pressure. Equipartition be-

$$\rho_{\mathrm{g,mp}}=\frac{\pi}{2\phi_{\mathrm{mp}}f_{g,P}}G\left(\frac{\Sigma_{\mathrm{g}}}{\sigma_{\mathrm{g}}}\right)^2.$$

$f_{g,P}$	0.5	Fractional contribution of gas self-gravity to mid-plane

Energy equilibrium

$$\mathcal{L} = \eta \frac{\Sigma_g(\sigma_g^2 - \sigma_{th}^2)}{H_g/\sqrt{\sigma_g^2 - \sigma_{th}^2}} \tag{25}$$

- Turbulent dissipation

$$= \frac{2(1+\beta)}{\pi G Q^2} \eta \phi_Q \phi_{nt}^{3/2} f_{g,Q}^2 \Omega^2 \sigma_g^3. \tag{26}$$

Thus if the star formation rate per unit area is $\dot{\Sigma}_*$, and the mean momentum injected per unit mass of stars formed is p_*/m_*, the rate of energy gain per unit area from star formation is

$$\mathcal{G} = \left\langle \frac{p_*}{m_*} \right\rangle \sigma_g \dot{\Sigma}_*. \tag{29}$$

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ture is assumed to
s a factor of order
corresponding to
all the energy being radiated in a single scale height-crossing time;

- Driving by star formation

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SFR

It is convenient to express the rate of star formation as

$$\dot{\Sigma}_* = \epsilon_{\rm ff} f_{\rm sf} \frac{\Sigma_{\rm g}}{t_{\rm ff}}. \tag{30}$$

Here $f_{\rm sf}$ is the fraction of the gas that is in a star-forming molecular phase rather than a warm atomic phase, and $t_{\rm ff}$ and $\epsilon_{\rm ff}$ are the free-fall time and star formation rate per free-fall time in this gas. As noted

$$\dot{\Sigma}_* = f_{\rm sf} \Sigma_{\rm g} \max\left(t_{\rm sf,T}^{-1}, t_{\rm sf,max}^{-1}\right), \tag{32}$$

where the first case is the Toomre regime and the second is the GMC regime. In terms of the galactic orbital period, the condition

'Toomre regime:

In our model we follow the approach set out in Forbes et al. (2014a), who base their model on the observations compiled by Krumholz et al. (2012). In this model, stars are assumed to form in a continuous medium with a free-fall time determined from $\rho_{\rm g,mp}$ as long as the resulting star formation time-scale,

$$t_{\rm sf,T} \equiv \frac{t_{\rm ff}}{\epsilon_{\rm ff}} = \frac{\pi Q}{4 f_{g,Q} \epsilon_{\rm ff}} \sqrt{\frac{3 f_{g,P} \phi_{\rm mp}}{2(1+\beta)}} \frac{1}{\Omega}, \tag{31}$$

GMC regime:
gas breaks up into individual molecular clouds whose densities are decoupled from the mean mid-plane density

$$t_{\rm sf,max} \approx 2 \text{ Gyr},$$

Radial transport

Our model is based on the realization that there is an alternative source of energy, radial transport. Such transport injects energy at scales comparable to the gas scale height, which then cascades down to become turbulent on smaller scales.

$$\frac{\partial \sigma_g}{\partial t} = \frac{\mathcal{G} - \mathcal{L}}{3\sigma_g \Sigma_g} + \frac{\sigma_g}{6\pi r \Sigma_g} \frac{\partial \dot{M}}{\partial r} + \frac{5(\partial \sigma_g / \partial r)}{6\pi r \Sigma} \dot{M} - \frac{1-\beta}{6\pi r^2 \Sigma_g \sigma_g} \Omega \mathcal{T}, \quad (36)$$

where $\mathcal{T}$ is the torque exerted by non-axisymmetric stresses, and

$$\dot{M} = -\frac{1}{v_\phi (1+\beta)} \frac{\partial \mathcal{T}}{\partial r} \quad (37)$$

If we search for solutions where that gas is in energy equilibrium, $\partial \sigma_g / \partial t = 0$, then equation (36) implies that

$$\frac{\sigma_g^2}{2\pi r} \frac{\partial \dot{M}}{\partial r} + \frac{5\sigma_g \dot{M}}{2\pi r} \frac{\partial \sigma_g}{\partial r} - \frac{1-\beta}{2\pi r^2} \Omega \mathcal{T} = \mathcal{L} - \mathcal{G}. \quad (38)$$

The critical velocity dispersion

The condition that $\mathcal{L} = \mathcal{G}$ for a marginally stable disc with $Q = Q_{\mathrm{min}}$ is satisfied if the gas velocity dispersion (total thermal plus non-thermal) is

$$\sigma_{\mathrm{g}} = \sigma_{\mathrm{sf}} \equiv \frac{4 f_{\mathrm{sf}} \epsilon_{\mathrm{ff}}}{\sqrt{3 f_{g,P}} \pi \eta \phi_{\mathrm{mp}} \phi_{Q} \phi_{\mathrm{nt}}^{3/2}} \left\langle \frac{p_*}{m_*} \right\rangle \times \max \left[1, \sqrt{\frac{3 f_{g,P}}{8(1+\beta)}} \frac{Q_{\mathrm{min}} \phi_{\mathrm{mp}}}{4 f_{g,Q} \epsilon_{\mathrm{ff}}} \frac{t_{\mathrm{orb}}}{t_{\mathrm{sf,max}}} \right] . \quad (39)$$

We find that, regardless of the value of fsf or various other parameters, our model predicts that the maximum velocity dispersion that can be sustained by star formation alone is $\sigma_{\mathrm{sf}} \approx 6$–$10$ km s−1. A corollary of this statement is that, if we observe a galaxy's velocity dispersion to be close to σ sf, we can conclude that the turbulence within it is primarily powered by star formation, whereas if we observe the velocity dispersion to be >> σ_{sf}, we can conclude that the turbulence is primarily powered by gravity.

The steady-state mass inflow rate

- With σ_{sf} defined, we are now in a position to calculate the mass inflow rate for galaxies with $\sigma_g > \sigma_{sf}$ and $Q = Q_{min}$.

$$\dot{M} = \dot{M}_{\mathrm{ss}} \equiv \frac{4(1+\beta)\eta\phi_Q\phi_{\mathrm{nt}}^{3/2}}{(1-\beta)GQ_{\mathrm{min}}^2} f_{g,Q}^2\sigma_{\mathrm{g}}^3\left(1-\frac{\sigma_{\mathrm{sf}}}{\sigma_{\mathrm{g}}}\right) \tag{49}$$

$$= 0.71 f_{g,Q,0.5}^2\sigma_{\mathrm{g},10}^3\,\mathrm{M}_\odot\ \mathrm{yr}^{-1} \cdot\left(1-\frac{\sigma_{\mathrm{th}}^2}{\sigma_{\mathrm{g}}^2}\right)^{3/2}\left(1-\frac{\sigma_{\mathrm{sf}}}{\sigma_{\mathrm{g}}}\right), \tag{50}$$

是否一定需要内流？

The other alternative is models without transport, which require that $\mathcal{G} = \mathcal{L}$. As noted above, this requirement can be satisfied in two ways. One is that we can keep the star formation law (equation 30) fixed. In the GMC regime we have $\mathcal{G} \propto \Sigma_{\rm g}\sigma_{\rm g}$ while $\mathcal{L} \propto \Sigma_{\rm g}^2\sigma_{\rm g}$, and thus $\mathcal{G} = \mathcal{L}$ is possible only for a single value of $\Sigma_{\rm g}$; since real galaxies clearly do not all have a single surface density, we discount this solution and instead focus on the Toomre regime. In the Toomre regime we have $\mathcal{G} = \mathcal{L}$ whenever $\sigma_{\rm g} = \sigma_{\rm sf}$ (equation 39). This implies that

$$Q = f_{g,Q}\frac{\kappa\sigma_{\rm sf}}{\pi G\Sigma_{\rm g}}$$

$$= \frac{8\sqrt{2(1+\beta)}}{\sqrt{3}\pi\eta\phi_{\rm mp}\phi_Q\phi_{\rm nt}^{3/2}} f_{\rm sf}\epsilon_{\rm ff}\left\langle\frac{p_*}{m_*}\right\rangle\frac{f_{g,Q}}{f_{g,P}^{1/2}G\Sigma_{\rm g}t_{\rm orb}} \tag{52}$$

$$= 3.6 f_{g,Q,0.5} f_{g,P,0.5}^{-1/2} t_{\rm orb,100}^{-1}\Sigma_{\rm g,10}^{-1}, \tag{53}$$

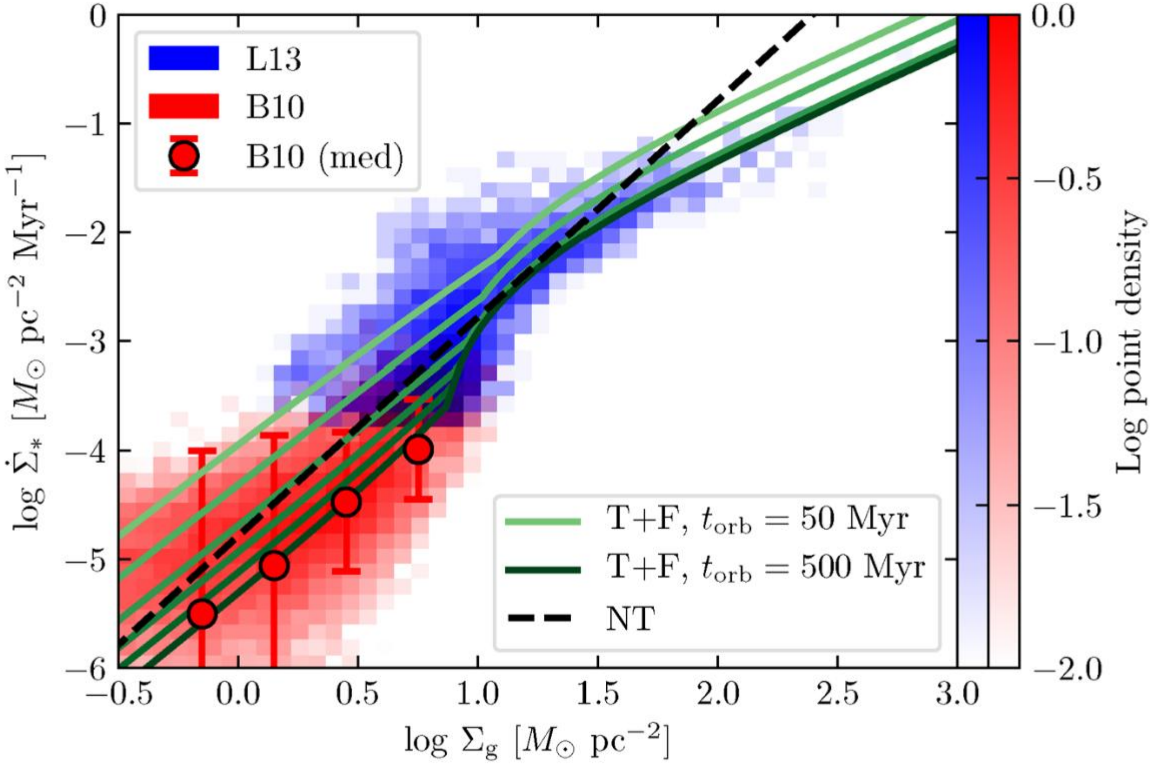
where $\Sigma_{\rm g,\,10} = \Sigma_{\rm g}/10\,{\rm M_\odot\,pc^{-2}}$. Thus if we do not include transport and keep the star formation law fixed, the model still predicts that $Q \approx 1$ for Solar Circle conditions ($\Sigma_{\rm g,10} \approx 1$, $t_{\rm orb,100} \approx 2$). However, for conditions like those found in ULIRGs ($\Sigma_{\rm g,10} \sim 100$, $t_{\rm orb,10} \sim 3$) or high-z star-forming discs ($\Sigma_{\rm g,10} \sim 10$, $t_{\rm orb,100} \sim 1$), the predicted value of Q is much smaller than unity.

Conversely, we can hold Q fixed and treat the quantity $f_{\mathrm{sf}}\epsilon_{\mathrm{ff}}$ as a free parameter, and use the relation $\mathcal{G} = \mathcal{L}$ to solve for it. In this case only the Toomre regime exists, and it is characterized by a star formation efficiency per free-fall time

$$\epsilon_{\mathrm{ff}} = \frac{\sqrt{3}\pi\,\eta\phi_{\mathrm{mp}}\phi_{Q}\phi_{\mathrm{nt}}^{3/2}}{4 f_{\mathrm{sf}}} \left\langle \frac{p_*}{m_*} \right\rangle^{-1} f_{g,P}^{-1/2}\sigma_{\mathrm{g}} \tag{54}$$

$$= 0.027 f_{\mathrm{sf}}^{-1} f_{g,P,0.5}^{-1/2}\sigma_{\mathrm{g},10}. \tag{55}$$

Thus ϵ_{ff} is $\sim$1 per cent for $\sigma_{\mathrm{g}} \approx 10\,\mathrm{km\,s^{-1}}$, but rises to $\gtrsim$10 per cent for the higher velocity dispersions typically seen in ULIRGs or high-redshift star-forming discs. Note that equation (54) is identical, up



L13
B10
B10 (med)
T+F, $t_{\rm orb} = 50$ Myr
T+F, $t_{\rm orb} = 500$ Myr
NT
$\log \dot{\Sigma}_*$ $[M_\odot$ pc^{-2} Myr$^{-1}]$
$\log \Sigma_{\rm g}$ $[M_\odot$ pc$^{-2}]$
Log point density
0
−1
−2
−3
−4
−5
−6
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.0
−0.5
−1.0
−1.5
−2.0