The *Prior*

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1, from *Likelihood* to *Posterior*

Bayesian Inference Method

$$P(\Theta|D) = \frac{P(D|\Theta) \times P(\Theta)}{P(D)}$$

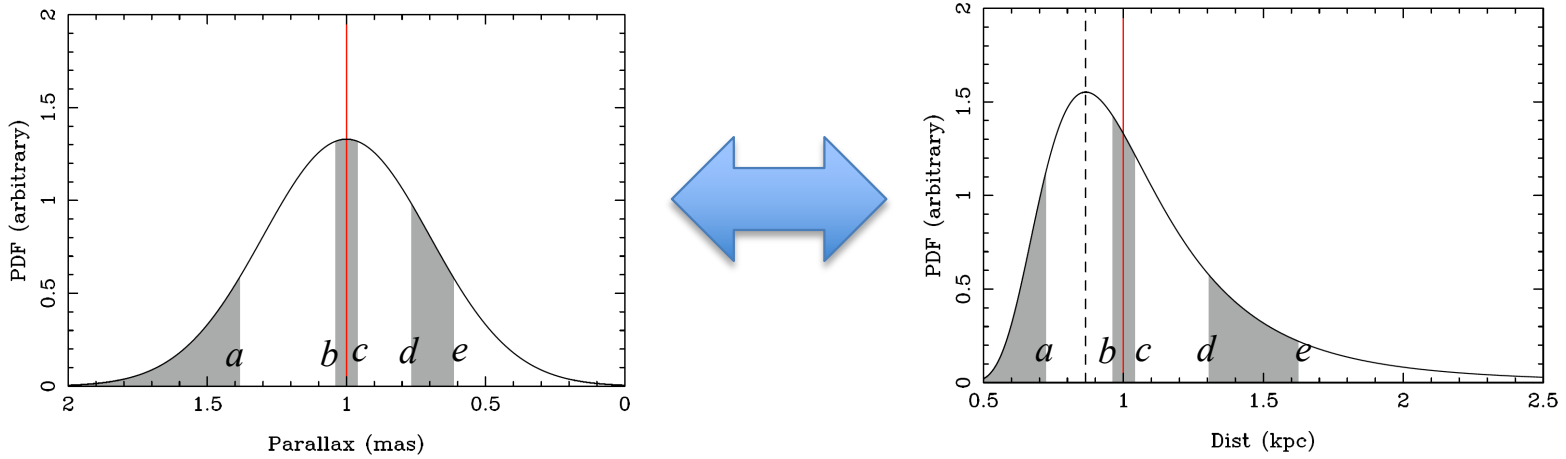
$$Post(\Theta|D) = \frac{L(D|\Theta) \times \pi(\Theta)}{Z(D)} \quad \Rightarrow \quad Post(\Theta|D) \propto L(D|\Theta) \times \pi(\Theta)$$

" the *prior*; it represents our state of *knowledge* (or *ignorance*) about the truth of the hypothesis before we have analyzed the *current* data. "

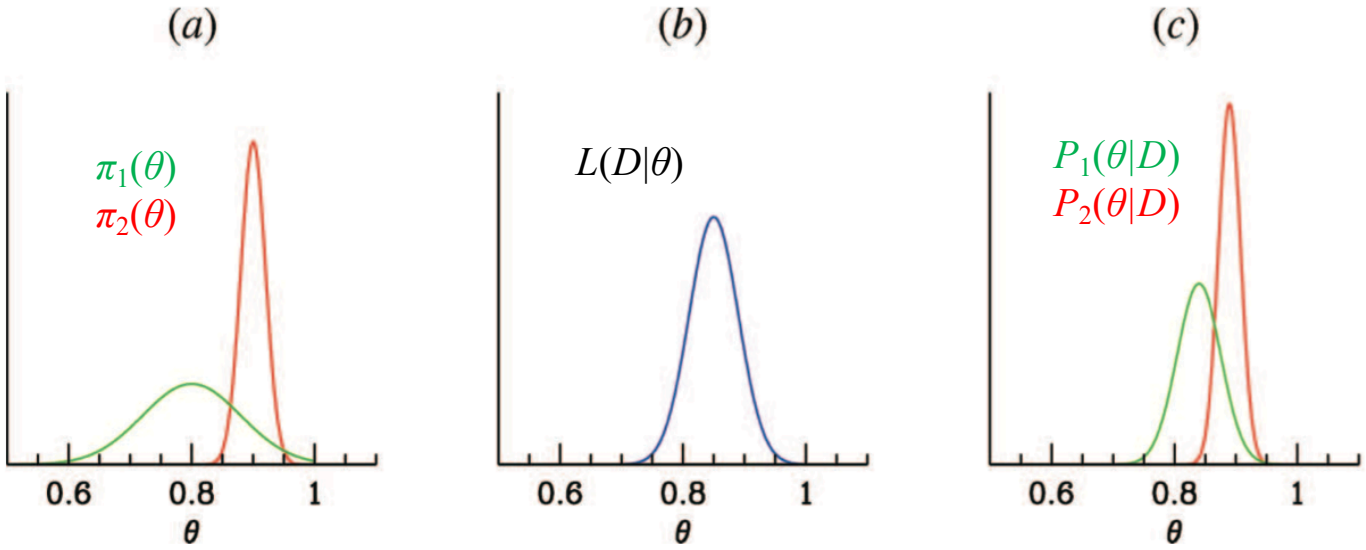
2, the Probability *density* function （PDF）

- The *posterior, likelihood* and *prior*, are *probability density* functions of parameter (***Θ***)
- The *density* functions are differential, and their integration are *probability*
  - ✓ it is more likely to have distance *at* '*b*' than *at* '*a*' .
  - ✓ the probability of distance *between* '*b*' to '*c*' is the same as the distance *nearer* than '*a*'.
  - ✗ the distance *at* '*b*' is more likely than that *less* than '*a*'.
- The *entire profile* (function) holds the *full information* of the parameter, not only the *peak* and the *dispersion*.

$$\varpi_{\max} = 1\ mas = 1.0kpc \quad \neq \quad D_{\max} = 0.865\ kpc$$



3, How the *prior* works ?



- *prior* and *likelihood* are competing on the constrain of the parameter

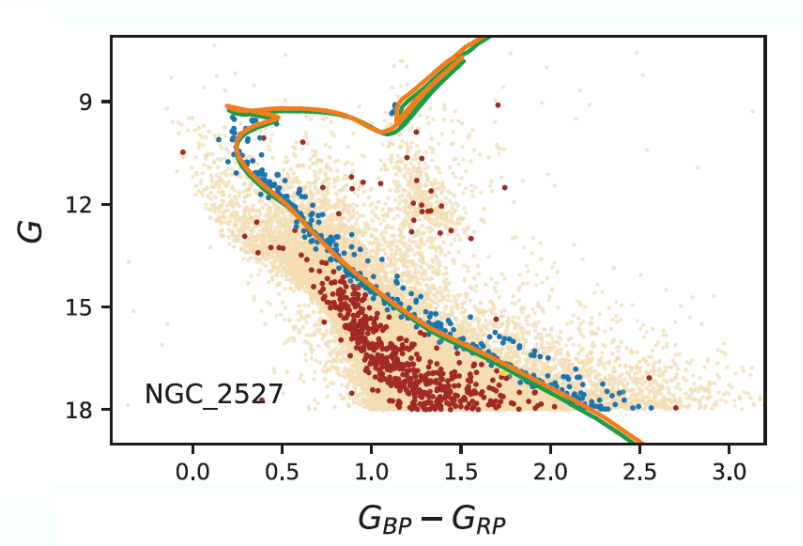
4, How to get the *prior*

1) The *posterior* from other data regression process

2) Physical consideration, subjective knowledge, the initial parameters in the L_{max} or χ^2 fitting, a fixed parameter, ……

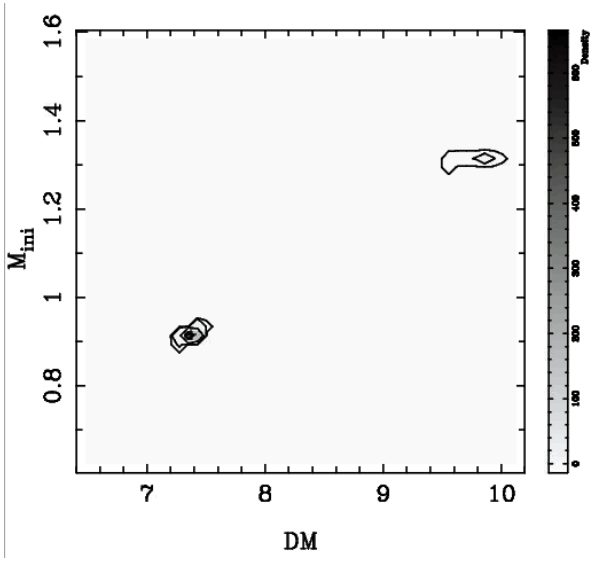
3) How about "*ignorance*" or "*no idea*" ? *flat prior*, or *uniform prior*, but it is practically shaped by the *chosen parameter*.

isochrone fitting of stellar cluster



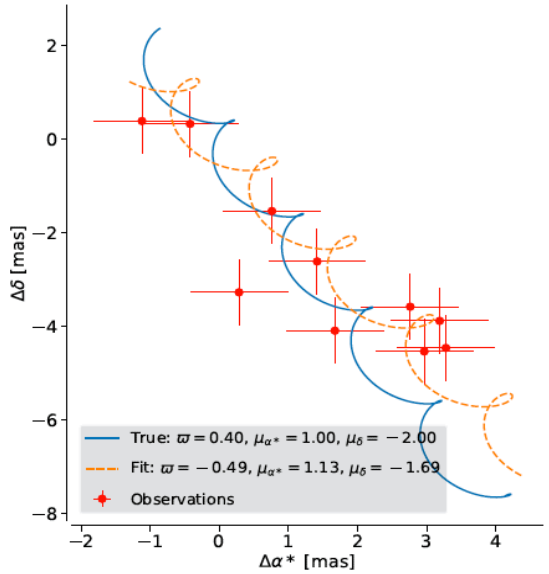
- *Age*
- *DM*
- *Av*
- Z_{met}

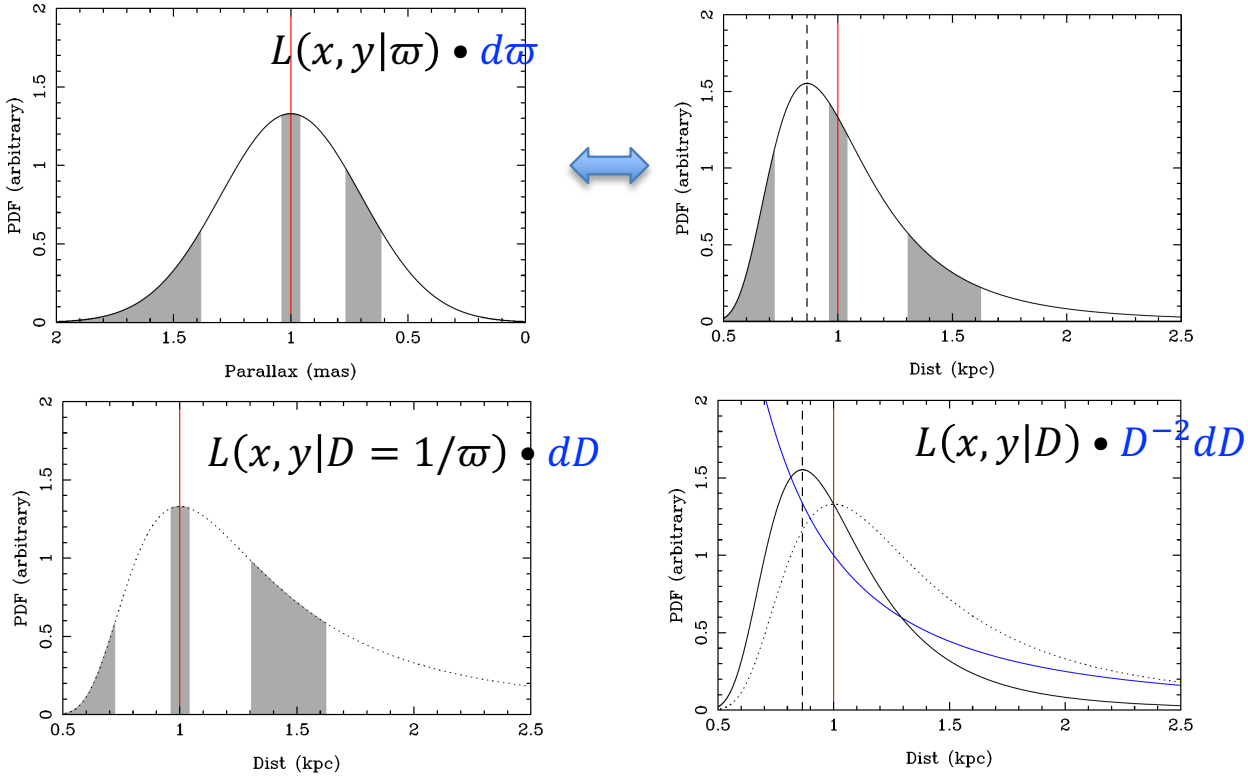
SED fitting of a star



5, "*no prior*" is a *prior*

$$[x(t), y(t)] \Rightarrow [\alpha, \delta, \mu_\alpha, \mu_\delta, \varpi]$$





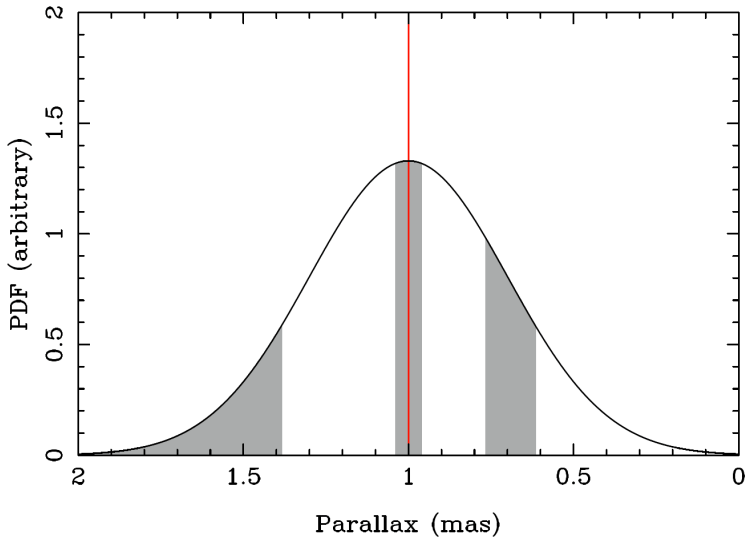
Why *Gaia* use ϖ, not the distance, as a parameter ?

The *data* directly describe the "*width*", and the PDF is theoretically Gaussian, based on the inference process.

from *Gaia* ϖ to the distance of a MW star

The prior of the MW structure:

a prior given by the distribution of stars along each line-of-sight as determined from a Galaxy model, which also accounted for interstellar extinction and the *Gaia* selection function.



$$P(r \mid L) = \begin{cases} \frac{1}{2L^3} r^2 e^{-r/L} & \text{if } r > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\phi_{\rm f}(\varpi) = \begin{cases} \frac{\varpi_L^3}{2(\varpi - \varpi_0)^4} \exp\left(\frac{-\varpi_L}{\varpi - \varpi_0}\right), & \text{if } \varpi > \varpi_0, \\ 0 & \text{otherwise} \end{cases} \qquad (3)$$

Remarks:

1, *prior* is one of the most important feature of Bayesian framework

2, *prior* is a kind of PDF

3, "*no prior*" is a prior. (*mag*., [*Fe*/*H*], $\lg M^*$, $\lg\sigma_v$ ……)